Day 274 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Conditional probability from a count (All boards)

50 students: 28 play football, 12 play tennis, 14 do neither.
(a) How many do both?
(b) Given a person who plays football, find P(this person plays tennis).

(a) At least one: $50 - 14 = 36$. Both $= 28 + 12 - 36 = 4$

(b) Of the 28 who play football, 4 also do the other: $P = \frac{4}{28} = \frac{1}{7}$

Answer: (a) 4 (b) $\frac{1}{7}$

2 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $\frac{(4x^3)^2 \times x^3}{8x^6}$.

$(4x^3)^2 = 16x^6$

Numerator $= 16x^6 \times x^3 = 16x^9$

Divide: $\frac{16}{8} = 2$ and $x^9 \div x^6 = x^3$

Answer: $2x^3$

3 Vectors in a triangle (proof and ratio) (All boards)

In triangle OAB, OA = a and OB = b. M lies on AB with AM : MB = 2 : 1, and N lies on OB with ON : NB = 2 : 3. Find OM and NM in terms of a and b.

$AB = b - a$, so $AM = \frac{2}{3}(b - a)$

$OM = a + AM = \frac{a}{3} + \frac{2b}{3}$

$ON = \frac{2b}{5}$, so $NM = OM - ON = \frac{a}{3} + \frac{4b}{15}$

Answer: OM $= \frac{a}{3} + \frac{2b}{3}$, NM $= \frac{a}{3} + \frac{4b}{15}$

4 Quartiles and outliers (AQA, Edexcel, OCR, MEI, Eduqas)

Ordered data, texts sent in a day by 7 students:
14, 16, 16, 23, 32, 33, 38.
Find Q_1, Q_3, the IQR and any outliers (more than 1.5 × IQR beyond a quartile).

$Q_1 = 16$, $Q_3 = 33$, IQR = 17

Lower limit $= 16 - 1.5 \times 17 = -\frac{19}{2}$

Upper limit $= 33 + 1.5 \times 17 = \frac{117}{2}$

No value lies outside these limits.
Answer: $Q_1 = 16$, $Q_3 = 33$, IQR = 17; no outliers

5 Position vectors and dividing a line in a ratio (All boards)

A has position vector $\begin{pmatrix}-2\\4\\2\end{pmatrix}$ and B has position vector $\begin{pmatrix}3\\-6\\-3\end{pmatrix}$.

The point P lies on AB with AP : PB = 2 : 3.

Find the position vector of P.

$\text{AB} = \begin{pmatrix}5\\-10\\-5\end{pmatrix}$

$\text{AP} = \frac{2}{5}\text{AB} = \begin{pmatrix}2\\-4\\-2\end{pmatrix}$

$\text{OP} = \text{OA} + \text{AP} = \begin{pmatrix}0\\0\\0\end{pmatrix}$

Answer: $\begin{pmatrix}\mathbf{0}\\\mathbf{0}\\\mathbf{0}\end{pmatrix}$

6 Rough inclined plane: sliding down (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 2 kg is released from rest on a rough plane inclined at angle α to the horizontal, where $\tan\alpha = \frac{5}{12}$.

The coefficient of friction is 0.2. Take g = 9.8 m s^{-2}. Find the acceleration down the plane, and the distance moved in 3 s.

$\cos\alpha = 12/13$, $\sin\alpha = 5/13$

$R = mg\cos\alpha$, $F = \mu R$

$ma = mg\sin\alpha - \mu mg\cos\alpha$, so $a = 9.8(5/13 - 0.2 \times 12/13)$

$s = \frac{1}{2}at^2 = \frac{1}{2} \times 1.96 \times 3^2$

Answer: 1.96 m s^{-2}; 8.82 m

7 3D vectors: magnitude, distance and unit vectors (All boards)

Points A and B have position vectors $\begin{pmatrix}0\\0\\-4\end{pmatrix}$ and $\begin{pmatrix}-10\\2\\-15\end{pmatrix}$.

Find AB and the distance AB.

$\text{AB} = \text{OB} - \text{OA} = \begin{pmatrix}-10\\2\\-11\end{pmatrix}$

$|\text{AB}| = \sqrt{10^2 + 2^2 + 11^2} = \sqrt{225} = 15$

Answer: AB $= \begin{pmatrix}\mathbf{-10}\\\mathbf{2}\\\mathbf{-11}\end{pmatrix}$**, distance 15**

8 Work-energy: sliding to rest (CIE)

A block of mass 4 kg is projected along a rough horizontal surface with speed 8.4 m s^{-1} and comes to rest after sliding 12 m.

Take g = 9.8 m s^{-2}.

Use work and energy to find the coefficient of friction.

Work done against friction = loss of KE

$\mu \times 4 \times 9.8 \times 12 = \frac{1}{2} \times 4 \times 8.4^2$

$470.4\,\mu = 141.12$

Answer: 0.3

9 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers x and y such that $x^2 - y^2 = 30$.

Assume $x^2 - y^2 = 30$, so $(x - y)(x + y) = 30$.
$x - y$ and $x + y$ have the same parity (their sum $2x$ is even).
If both are odd, the product is odd; if both are even, the product is a multiple of 4.
But 30 is even and is not a multiple of 4 ($30 = 4 \times 7 + 2$), a contradiction.
So no such integers exist.
Answer: no integer solutions (product is odd or a multiple of 4)

10 Perpendicular bisector (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is the point $(-2, -6)$ and B is the point $(5, 1)$.
Find the perpendicular bisector of AB as $ax + by + c = 0$, with integer a, b, c.

Midpoint of AB = $(\frac{3}{2}, -\frac{5}{2})$

Gradient of AB = $\frac{7}{7} = 1$

Perpendicular gradient = -1

$y + \frac{5}{2} = -(x - \frac{3}{2})$, which gives $x + y + 1 = 0$

Answer: $x + y + 1 = 0$