Day 273 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Locating roots: change of sign (All boards)

The function $f(x) = e^x - 6x$.
Show that $f(x) = 0$ has a root between $x = 2.5$ and $x = 3$.

$f(2.5) = -2.82$
$f(3) = 2.09$
f is continuous and changes sign, so there is a root in the interval
Answer: f(2.5) < 0 and f(3) > 0: sign change

2 Binomial probabilities (All boards)

$X \sim B(10, 0.5)$. Find, to 3 significant figures,
(a) $P(1 \leq X \leq 4)$
(b) $P(X > 3)$

(a) $P(1 \leq X \leq 4) = P(X \leq 4) - P(X \leq 0)$
(a) = 0.376
(b) $P(X > 3) = 1 - P(X \leq 3)$
(b) = 0.828
Answer: (a) 0.376 (b) 0.828

3 Reciprocal graph: asymptotes and intercepts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $y = \frac{-2}{x + 4} + 1$.
(a) Write down the asymptotes.
(b) Find where the curve crosses the axes.

(a) The denominator is zero when $x = -4$, and $y \to 1$ as $x \to \pm\infty$
Asymptotes: $x = -4$ and $y = 1$
(b) $x = 0$: $y = -\frac{1}{2} + 1 = \frac{1}{2}$
$y = 0$: $\frac{-2}{x + 4} = -1$, so $x + 4 = 2$, $x = -2$
Answer: (a) $x = -4$, $y = 1$ (b) $(0, \frac{1}{2})$ and $(-2, 0)$

4 Exact values of sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\cot\theta = \frac{4}{3}$ and θ is acute,
find the exact value of $\sec\theta$.

Use $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \operatorname{cosec}^2\theta$
$\tan\theta = \frac{3}{4}$, so $\sec^2\theta = 1 + \frac{9}{16} = \frac{25}{16}$
$\sec\theta = \frac{5}{4}$ (positive as θ is acute)
Answer: $\frac{5}{4}$

5 Proof by deduction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that the sum of 3 consecutive even integers is always a multiple of 3.

Let the integers be $2n$, $2n + 2$, $2n + 4$
Sum = $6n + 6$
$= 3(2n + 2)$
n is an integer, so $(2n + 2)$ is an integer, so the sum is a multiple of 3.
Answer: sum = 3(2n + 2), a multiple of 3

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6 Constant acceleration, two stages of working (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car moving in a straight line with constant acceleration increases its speed from 12 m s^{-1} to 30 m s^{-1} while covering 189 m.
Find (a) the acceleration, (b) the time taken.

(a) $v^2 = u^2 + 2as$: $900 = 144 + 378\,a$, so $a = 2$
(b) $v = u + at$: $30 = 12 + 2t$
$t = 18 \div 2 = 9$
Answer: (a) 2 m s^{-2} (b) 9 s

7 Binomial one-tailed test (All boards)

A manufacturer claims that a proportion 0.3 of its sweets are red. A sample of 25 sweets has 12 red.
Test, at the 1% level, whether the true proportion is higher than claimed.

H_0: $p = 0.3$, H_1: $p > 0.3$. Let $X \sim B(25, 0.3)$ under H_0
$P(X \geq 12) = 1 - P(X \leq 11) = 0.0442$
$0.0442 > 0.01$, so do not reject H_0
There is insufficient evidence that the proportion is higher than claimed
Answer: do not reject H_0: insufficient evidence that the proportion is higher

8 Trapezium rule (AQA, Edexcel, OCR, MEI, Eduqas)

Values of y = f(x) are shown.
x: 0, 0.5, 1, 1.5, 2, 2.5
y: 3, 1.3, 1, 3, 2.1, 2.8

Estimate $\int_0^{2.5} f(x)\,dx$ using the trapezium rule.

$h = 0.5$
Area $\approx \frac{h}{2}[y_0 + y_5 + 2(y_1 + \ldots + y_4)]$
$= \frac{0.5}{2}[3 + 2.8 + 2(1.3 + 1 + 3 + 2.1)]$
$= 5.15$
Answer: 5.15

9 Maximum speed from acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t s its velocity is $v = 3t - 0.5t^2$ m s^{-1}, for $t \geq 0$.
Find the maximum speed of the particle and the time at which it occurs.

$a = dv/dt = 3 - t$
Maximum speed when $a = 0$: $t = 3 \div 1 = 3$
$v = 3 \times 3 - 0.5 \times 3^2$
Answer: 4.5 m s^{-1} at t = 3 s

10 Partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{-x + 14}{(x + 1)(x - 4)}$ in partial fractions.

$\frac{-x + 14}{(x + 1)(x - 4)} \equiv \frac{A}{(x + 1)} + \frac{B}{(x - 4)}$, so $-x + 14 \equiv A(x - 4) + B(x + 1)$
$x = -1$: $15 = A \times (-5)$, so $A = -3$
$x = 4$: $10 = B \times 5$, so $B = 2$
Answer: $-\frac{3}{(x + 1)} + \frac{2}{(x - 4)}$