Day 272 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Quadratic equations in sin x, cos x, tan x (All boards)

Solve $4\cos^2 x + 3 - 7\cos x = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p. where necessary.

$(4\cos x - 3)(\cos x - 1) = 0$, so $\cos x = \frac{3}{4}$ or $\cos x = 1$

$\cos x = \frac{3}{4}$: $x = 41.4°, 318.6°$

$\cos x = 1$: $x = 0.0°, 360.0°$

Answer: x = 0.0°, 41.4°, 318.6°, 360.0°

2 Scatter diagram (AQA, Edexcel, OCR, MEI, Eduqas)

The scatter diagram shows eight pairs of values of x and y.
(a) Describe the correlation.
(b) Why should $y = 4 + x$ not be used for $x = 15$?

(a) The points lie loosely around a straight line rising: weak positive correlation
(b) The data cover x = 1 to 10; x = 15 is outside this range
so using the line would be extrapolation and may be unreliable
Answer: (a) weak positive correlation (b) extrapolation outside the range of the data

3 Arithmetic sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of all the multiples of 7 that lie between 139 and 442.

First multiple = $7 \times 20 = 140$, last multiple = $7 \times 63 = 441$
This is an arithmetic series with d = 7 and 44 terms

$S = \frac{44}{2}(140 + 441) = 12782$

Answer: 12782

4 Braking distance and time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist travelling at 20 m s^{-1} brakes and comes to rest with constant deceleration 2.5 m s^{-2}.
Find (a) the distance travelled while braking, (b) the time taken to stop.

(a) $0 = 20^2 - 2 \times 2.5 \times s$
$s = 400 \div 5 = 80$
(b) $0 = 20 - 2.5t$, so $t = 20 \div 2.5$
Answer: (a) 80 m (b) 8 s

5 Collinear points and parallel vectors (All boards)

Points A, B and C have position vectors $\begin{pmatrix}3\\3\\2\end{pmatrix}$,

$\begin{pmatrix}0\\6\\1\end{pmatrix}$ and $\begin{pmatrix}-6\\12\\r\end{pmatrix}$ respectively.

Given that A, B, C are collinear, find r.

$\overrightarrow{AB} = \begin{pmatrix}-3\\3\\-1\end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix}-9\\9\\r-2\end{pmatrix}$

$\overrightarrow{AC} = \lambda\overrightarrow{AB}$: component 1 gives $-9 = \lambda \times -3$, so $\lambda = 3$
$r - 2 = 3 \times -1$, so $r = -1$
Answer: r = −1

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6 Coefficient in a product with a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^2 in the expansion of
$(3 - x)(1 + 3x)^5$.

$(1 + 3x)^5$ has x^1 coefficient 15 and x^2 coefficient 90
x^2 term of the product = $3 \times 90 + (-1) \times 15$
$= 255$
Answer: 255

7 Equations with sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2\tan^2 x + 9\sec x + 11 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p.

Use $\tan^2 x = \sec^2 x - 1$: $2\sec^2 x + 9\sec x + 9 = 0$
$(\sec x + 3)(2\sec x + 3) = 0$, so $\sec x = -3$ or $-\frac{3}{2}$
$\sec x = -3$, so $\cos x = -\frac{1}{3}$: $x = 109.5°, 250.5°$
$\sec x = -\frac{3}{2}$, so $\cos x = -\frac{2}{3}$: $x = 131.8°, 228.2°$
Answer: x = 109.5°, 131.8°, 228.2°, 250.5°

8 Discriminant problems (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Show that the equation $3x^2 + 5x + 12 = 0$ has no real roots.

$b^2 - 4ac = 5^2 - 4 \times 3 \times 12$
$= 25 - 144 = -119$
The discriminant is negative, so there are no real roots.
Answer: $b^2 - 4ac = -119 < 0$

9 Pulley system: speed on reaching the ground (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (4 kg) and B (6 kg) are joined by a light string over a smooth pulley. Both are held with the string taut, B is 0.5 m above the ground, then released.
Take $g = 9.8$ m s^{-2}.
Find the speed with which B hits the ground.

$a = (6 - 4)g \div (6 + 4) = 1.96$
$v^2 = 0 + 2 \times 1.96 \times 0.5 = 1.96$
$v = 1.4$
Answer: 1.4 m s^{-1}

10 Histogram bar heights (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A class of width 10, frequency 40, is a bar 5 cm wide, 2 cm tall.
(a) Find the bar height for width 10, frequency 24.
(b) Find the frequency of a 1 cm by 3 cm bar.

Area of bar $\propto$ frequency: $5 \times 2 = 10$ cm^2 represents 40
so 1 unit of frequency = 0.25 cm^2
(a) Width = 5 cm, area = $0.25 \times 24 = 6$ cm^2, height = $6 \div 5 = 1.2$ cm
(b) Area = $1 \times 3 = 3$ cm^2, frequency = $3 \div 0.25 = 12$
Answer: (a) 1.2 cm (b) 12