Day 271 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Perpendicular lines and shortest distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line l has equation $y = -\frac{1}{2}x + 3$. The point P is (0, 8).

(a) Find the line through P perpendicular to l, as $ax + by + c = 0$ with integer a, b, c.
(b) Find the foot F and the exact distance PF.

(a) Gradient of l $= -\frac{1}{2}$, so the perpendicular has gradient 2

$y - 8 = 2x$, giving $2x - y + 8 = 0$

(b) Solve $y = -\frac{1}{2}x + 3$ with $2x - y + 8 = 0$: F = $(-2, 4)$

$PF^2 = 2^2 + 4^2 = 20$

Answer: (a) $2x - y + 8 = 0$ (b) F$(-2, 4)$, PF $= 2\sqrt{5}$

2 Rough table and a hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 9 kg lies on a rough horizontal table (coefficient of friction 0.2). It is joined by a light string over a smooth pulley to a particle of mass 5 kg hanging freely.

Take g = 9.8 m s^{-2}.

The system is released. Find the acceleration and the tension in the string.

Block: $T - 0.2 \times 9\,g = 9\,a$
Hanging: $5\,g - T = 5\,a$
Add: $5\,g - 1.8\,g = 14\,a$
$a = 2.24$
$T = 5(9.8 - 2.24)$

Answer: a = 2.24 m s^{-2}; T = 37.8 N

3 Coded data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

7 values of x are coded using $y = \frac{(x - 20)}{2}$.

$\Sigma y = 5$ and $\Sigma y^2 = 109$.

Find the mean and standard deviation of x (3 s.f.).

Mean of $y = 5 \div 7 = 0.7143$
Mean of $x = 20 + 2 \times 0.7143 = 21.4$

Variance of $y = \frac{109}{7} - (0.7143)^2 = 15.06$

SD of y = 3.881, so SD of x $= 2 \times 3.881 = 7.76$

Answer: mean 21.4, standard deviation 7.76

4 3D vectors: magnitude, distance and unit vectors (All boards)

The vector $\mathbf{a} = \begin{pmatrix} 10 \\ 2 \\ -11 \end{pmatrix}$.

Find $|\mathbf{a}|$ and a unit vector in the direction of a.

$|\mathbf{a}| = \sqrt{10^2 + 2^2 + 11^2} = \sqrt{225} = 15$

Unit vector $= \frac{1}{15}\mathbf{a}$

Answer: $|\mathbf{a}| = 15$, unit vector $\frac{1}{15}\begin{pmatrix} 10 \\ 2 \\ -11 \end{pmatrix}$

5 Modulus equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|3x - 6| = -4x - 6$.

Case 1: $3x - 6 = -4x - 6$, so $x = 0$
Case 2: $3x - 6 = -(-4x - 6)$, so $x = -12$
Check $x = 0$: $|3x - 6| = 6$, RHS $= -6 < 0$, rejected
Check $x = -12$: $|3x - 6| = 42$, RHS = 42

Answer: x = −12

6 Chain rule (All boards)

Given that $y = \ln(4x^2 + 7)$,

find $\frac{dy}{dx}$, and the gradient of the curve when $x = 1$.

Chain rule: differentiate the outer function, then multiply by the derivative of the inner function

$\frac{dy}{dx} = \frac{8x}{4x^2 + 7}$

When $x = 1$: gradient $= \frac{8}{11}$

Answer: $\frac{dy}{dx} = \frac{8x}{4x^2 + 7}$; gradient $\frac{8}{11}$

7 Two particles over a pulley (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles of mass 3 kg and 11 kg hang on a light string over a smooth fixed pulley, as shown, and are released from rest.
Take $g = 9.8$ m s^{-2}.
Find the acceleration and the tension in the string.

Heavier: $11g - T = 11a$
Lighter: $T - 3g = 3a$
Add: $8g = 14a$, so $a = 5.6$
$T = 3(9.8 + 5.6)$
Answer: $a = 5.6$ m s^{-2}; $T = 46.2$ N

8 Reduction to linear form (logarithms) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The variables x and y satisfy $y = Ae^{kx}$. A graph of ln y against x is a straight line through (0, 2) and (4, 3.6).
Find A to 3 s.f. and k.

$\ln y = \ln A + kx$, so $\ln A = 2$ and $A = e^2 = 7.389$

k = gradient $= \frac{3.6 - 2}{4 - 0} = 0.4$

Answer: A = 7.39, k = 0.4

9 Tangent lines of a given gradient (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $5x - 12y = k$ is a tangent to the circle
$(x - 4)^2 + y^2 = 169$.
Find the two possible values of k.

The radius to the point of contact is perpendicular to the line, so it is parallel to $(5, -12)$.
A unit step in this direction is $(5, -12)/13$, so the points of contact are
$(4, 0) \pm 13 \times (5, -12)/13 = (4, 0) \pm (5, -12)$
T = $(9, -12)$ or $(-1, 12)$, and $k = 5x - 12y$ gives:
$k = 189$ or $k = -149$
Answer: k = 189 or k = −149

10 Regression line (AQA, Edexcel, OCR, MEI, Eduqas)

For 8 pairs of values, $\bar{x} = 6$, $\bar{y} = 40$, $S_{xx} = 100$ and $S_{xy} = 250$.
(a) Find the regression line of y on x in the form $y = a + bx$.
(b) Estimate y when $x = 3$.

(a) $b = \frac{S_{xy}}{S_{xx}} = \frac{250}{100} = 2.5$

$a = \bar{y} - b\bar{x} = 40 - 2.5 \times 6 = 25$
$y = 25 + 2.5x$
(b) $y = 25 + 2.5 \times 3 = 32.5$
Answer: y = 25 + 2.5x; y = 32.5