Day 270 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Quadratic equations in sin x, cos x, tan x (All boards)

Solve $4\tan^2 x - 4\tan x - 3 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p. where necessary.

$(2\tan x + 1)(2\tan x - 3) = 0$, so $\tan x = -\frac{1}{2}$ or $\tan x = \frac{3}{2}$

$\tan x = -\frac{1}{2}$: $x = 153.4°, 333.4°$

$\tan x = \frac{3}{2}$: $x = 56.3°, 236.3°$

Answer: x = 56.3°, 153.4°, 236.3°, 333.4°

2 Differentiating displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves on a straight line. Its displacement from O at time t seconds is $s = 4t^3 - 4t^2 + 8t$ metres.
Find its velocity and acceleration when t = 4.

$v = ds/dt = 12t^2 - 8t + 8$
$a = dv/dt = 24t - 8$
$t = 4$: $v = 168$, $a = 88$
Answer: $v = 168\ \text{m s}^{-1}$, $a = 88\ \text{m s}^{-2}$

3 Integrating composite functions (reverse chain rule) (All boards)

Find (a) $\int (5x + 5)^5\, dx$,

(b) the exact value of $\int_0^2 (5x + 5)^5\, dx$.

(a) Reverse the chain rule and divide by the coefficient of x: $\frac{(5x + 5)^6}{30} + c$

(b) $\left[\frac{(5x + 5)^6}{30}\right]_0^2$
$= \frac{1137500}{3}$

Answer: (a) $\frac{(5x + 5)^6}{30} + c$ (b) $\frac{1137500}{3}$

4 Regression line (AQA, Edexcel, OCR, MEI, Eduqas)

For 12 pairs of values, $\bar{x} = 12$, $\bar{y} = 25$, $S_{xx} = 20$ and $S_{xy} = -20$.
(a) Find the regression line of y on x in the form $y = a + bx$.
(b) Estimate y when x = 10.

(a) $b = \frac{S_{xy}}{S_{xx}} = \frac{-20}{20} = -1$
$a = \bar{y} - b\bar{x} = 25 - -1 \times 12 = 37$
$y = 37 - x$
(b) $y = 37 + (-1) \times 10 = 27$
Answer: $y = 37 - x$; $y = 27$

5 Proof by deduction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that the sum of 7 consecutive odd integers is always a multiple of 7.

Let the integers be $2n + 1$, $2n + 3$, $2n + 5$, $2n + 7$, $2n + 9$, $2n + 11$, $2n + 13$
Sum $= 14n + 49$
$= 7(2n + 7)$
n is an integer, so $(2n + 7)$ is an integer, so the sum is a multiple of 7.
Answer: sum = 7(2n + 7), a multiple of 7

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6 Particle on a table joined to a hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 10 kg lies on a smooth horizontal table. It is attached by a light string over a smooth pulley at the edge of the table to a particle of mass 10 kg hanging freely.

Take $g = 9.8$ m s^{-2}.

The system is released from rest. Find the acceleration and the tension in the string.

Hanging: $10g - T = 10a$

Table: $T = 10a$

$10g = 20a$, so $a = 4.9$

$T = 10 \times 4.9$

Answer: $a = 4.9$ m s^{-2}; $T = 49$ N

7 Cubic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x(x + 3)(x + 4) \leq 0$.

Critical values: $x = -4$, $x = -3$, $x = 0$

The cubic has a positive x^3 term, so the graph goes from bottom left to top right and crosses the axis at each critical value.

Reading from the sketch: $x \leq -4$ or $-3 \leq x \leq 0$

Answer: $x \leq -4$ or $-3 \leq x \leq 0$

8 Scalar product and angle between vectors (CIE)

$\mathbf{a} = \begin{pmatrix} -2 \\ 0 \\ -4 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix}$.

Find the angle between a and b, to 1 d.p.

$\mathbf{a} \cdot \mathbf{b} = (-2)(-2) + (0)(1) + (-4)(-4) = 20$

$|\mathbf{a}| = \sqrt{20}$, $|\mathbf{b}| = \sqrt{21}$

$\cos\theta = \dfrac{20}{\sqrt{20}\sqrt{21}} = 0.9759$

$\theta = 12.6°$

Answer: 12.6°

9 Conditional probability from a count (All boards)

40 members: 20 swim, 11 cycle, 18 do neither.

(a) How many do both?

(b) Given a person who swims, find P(this person cycles).

(a) At least one: $40 - 18 = 22$. Both $= 20 + 11 - 22 = 9$

(b) Of the 20 who swim, 9 also do the other: $P = \dfrac{9}{20} = \dfrac{9}{20}$

Answer: (a) 9 (b) $\dfrac{9}{20}$

10 Parametric curve meeting a line (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = 3t + 3$, $y = t^2 + 6t + 1$.

The line $y = 3x - 4$ meets the curve at two points.

Find the coordinates of these points.

Substitute x and y into the line: $t^2 + 6t + 1 = 9t + 5$

$t^2 - 3t - 4 = 0$, so $(t - 4)(t + 1) = 0$

$t = -1$ or $t = 4$, giving the points $(0, -4)$ and $(15, 41)$

Answer: $(0, -4)$ and $(15, 41)$