Day 27 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Equation of a circle from given information (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has centre $(-5, 1)$ and passes through the point $(4, -6)$.
Find an equation of the circle.

$r^2 = (4 - (-5))^2 + (-6 - 1)^2$
$= 81 + 49 = 130$
Answer: $(x + 5)^2 + (y - 1)^2 = 130$

2 Comparing box plots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The box plots show the marks in a mock exam for two groups.
(a) Find the interquartile range of each group.
(b) Compare the two distributions.

(a) Class A: $60 - 40 = 20$. Class B: $25 - 15 = 10$
(b) Medians: 45 (Class A) and 20 (Class B), so Class A has the higher median
IQRs: Class A has the larger IQR, so its scores are more spread out
Answer: (a) 20 and 10 (b) Class A has the higher median; Class A is more spread out (larger IQR), Class B more consistent

3 R cos(θ − α) form: express and maximum (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $4 \sin\theta - 3 \cos\theta$ in the form $R \sin(\theta - \alpha)$, with $R > 0$ and $0 < \alpha < 90°$.
Hence state the minimum value of the expression. Give α to 1 d.p.

$R \cos\alpha = 4$, $R \sin\alpha = 3$

$R = 5$ and $\tan\alpha = \frac{3}{4}$, so $\alpha = 36.9°$

Minimum value of $R \sin(\theta - \alpha)$ is $-R$
Answer: $5 \sin(\theta - 36.9°)$; minimum -5

4 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $\frac{(3x^2)^2 \times 2x^2}{9x^7}$.

$(3x^2)^2 = 9x^4$

Numerator $= 9 \times 2x^6 = 18x^6$

Divide: $\frac{18}{9} = 2$ and $x^6 \div x^7 = x^{-1} = \frac{1}{x}$

Answer: $\frac{2}{x}$

5 Quadratic equations in sin x, cos x, tan x (All boards)

Solve $2 \sin^2 x + \sin x - 1 = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p. where necessary.

$(2 \sin x - 1)(\sin x + 1) = 0$, so $\sin x = \frac{1}{2}$ or $\sin x = -1$

$\sin x = \frac{1}{2}$: $x = 30.0°, 150.0°$

$\sin x = -1$: $x = 270.0°$
Answer: $x = 30.0°, 150.0°, 270.0°$

6 Velocity parallel to a direction (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves so that at time t s its velocity is $\mathbf{v} = (3t^2 + 5)\mathbf{i} + (5t - 15)\mathbf{j}$ m s^{-1}.
Find the time when the particle is moving in the direction of $\mathbf{i}$, and its speed then.

Parallel to $\mathbf{i}$: $\mathbf{j}$ component $= 0$
$5t - 15 = 0$, so $t = 3$
$\mathbf{v} = (3 \times 3^2 + 5)\mathbf{i} = 32\mathbf{i}$
Answer: $t = 3$ s, speed 32 m s^{-1}

7 Partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{-2x^2 - 29x - 59}{(x-2)(x+3)^2}$ in partial fractions.

$\frac{-2x^2 - 29x - 59}{(x-2)(x+3)^2} \equiv \frac{A}{(x-2)} + \frac{B}{(x+3)} + \frac{C}{(x+3)^2}$

$-2x^2 - 29x - 59 \equiv A(x+3)^2 + B(x-2)(x+3) + C(x-2)$

$x = 2$ gives $A = -5$; $x = -3$ gives $C = -2$

Coefficients of x^2: $-2 = A + B$, so $B = 3$

Answer: $\mathbf{-\frac{5}{(x-2)} + \frac{3}{(x+3)} - \frac{2}{(x+3)^2}}$

8 Rough inclined plane: sliding down (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 3 kg is released from rest on a rough plane inclined at angle α to the horizontal, where $\tan\alpha = \frac{5}{12}$.

The coefficient of friction is 0.2. Take $g = 9.8$ m s^{-2}. Find the acceleration down the plane, and the distance moved in 2 s.

$\cos\alpha = 12/13$, $\sin\alpha = 5/13$

$R = mg\cos\alpha$, $F = \mu R$

$ma = mg\sin\alpha - \mu mg\cos\alpha$, so $a = 9.8(5/13 - 0.2 \times 12/13)$

$s = \frac{1}{2}at^2 = \frac{1}{2} \times 1.96 \times 2^2$

Answer: 1.96 m s^{-2}; 3.92 m

9 Finding mean and standard deviation from two probabilities (All boards)

The random variable X is normally distributed.
P(X < 20) = 0.1587 and P(X > 45) = 0.0668.
Find the mean and standard deviation (3 s.f.).

$z_1 = -0.9998$ and $z_2 = 1.500$

$20 = \mu + (-0.9998)\sigma$ and $45 = \mu + 1.500\sigma$

Subtract: $25 = 2.500\,\sigma$, so $\sigma = 10.0$

$\mu = 20 - (-0.9998)(10.00) = 30.0$

Answer: $\mu = 30.0$, $\sigma = 10.0$

10 Double angle values from a given ratio (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\tan\theta = -\frac{5}{12}$ and $90° < \theta < 180°$,

find the exact value of $\sin 2\theta$.

Right-angled triangle 5, 12, 13: $\sin\theta = \frac{5}{13}$, $\cos\theta = -\frac{12}{13}$ (θ obtuse)

$\sin 2\theta = 2\sin\theta\cos\theta = 2 \times \frac{5}{13} \times (-\frac{12}{13}) = -\frac{120}{169}$

Answer: $\mathbf{-\frac{120}{169}}$