Day 269 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Equilibrium of three forces (components) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is in equilibrium under three forces: P N at angle θ above the positive x-axis, a force of 30 N in the negative x-direction and a force of Q N in the negative y-direction.

$\cos\theta = \frac{15}{17}$. Find P and Q.

Resolve horizontally: $P\cos\theta = 30$, so $P \times \frac{15}{17} = 30$

$P = 34$

Resolve vertically: $Q = P\sin\theta = 34 \times \frac{8}{17}$

Answer: P = 34 N; Q = 16 N

2 3D vectors: magnitude, distance and unit vectors (All boards)

The vector $\mathbf{a} = \begin{pmatrix} 1 \\ 12 \\ 12 \end{pmatrix}$.

Find $|\mathbf{a}|$ and a unit vector in the direction of a.

$|\mathbf{a}| = \sqrt{1^2 + 12^2 + 12^2} = \sqrt{289} = 17$

Unit vector $= \frac{1}{17}\mathbf{a}$

Answer: $|\mathbf{a}| = 17$, unit vector $\frac{1}{17}\begin{pmatrix} 1 \\ 12 \\ 12 \end{pmatrix}$

3 Recurrence relations (AQA, Edexcel, OCR, MEI, Eduqas)

A sequence is defined by $u_{n+1} = k\,u_n + 6$, $u_1 = 2$.

Given that $u_3 = 6$, find the possible values of k.

$u_2 = 2k + 6$

$u_3 = k(2k + 6) + 6 = 2k^2 + 6k + 6$

$2k^2 + 6k + 6 = 6$, so $2k^2 + 6k = 0$

$2k(k + 3) = 0$

$k = -3$ or $k = 0$

Answer: k = −3 or k = 0

4 Standard integrals (definite) (All boards)

Find the exact value of

$\int_0^{\frac{\pi}{2}} (4\sin 2x + 3\cos 2x)\,dx$.

$\int (4\sin 2x + 3\cos 2x)\,dx = \frac{3\sin 2x}{2} - 2\cos 2x$

$= \left[\frac{3\sin 2x}{2} - 2\cos 2x\right]_0^{\frac{\pi}{2}}$

$= 4$

Answer: 4

5 Binomial one-tailed test (All boards)

A manufacturer claims that a proportion 0.25 of its sweets are red. A sample of 30 sweets has 14 red.

Test, at the 5% level, whether the true proportion is higher than claimed.

H_0: $p = 0.25$, H_1: $p > 0.25$. Let $X \sim B(30, 0.25)$ under H_0

$P(X \geq 14) = 1 - P(X \leq 13) = 0.0082$

$0.0082 < 0.05$, so reject H_0

There is sufficient evidence that the proportion is higher than claimed

Answer: reject H_0: evidence that the proportion is higher

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6 Triangle formed by two lines and an axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Line l_1 has equation $2x + 3y = 10$ and line l_2 has equation $2x + y = 6$.
The lines meet at P. Find the area of the triangle formed by l_1, l_2 and the x-axis.

Solve simultaneously: P = (2, 2)
l_1 meets $y = 0$ at $x = 5$, and l_2 meets $y = 0$ at $x = 3$
Base = 2, height = 2
Area = $\frac{1}{2}$ × base × height = 2

Answer: 2

7 Projection from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards at 4.9 m s^{-1} from a point 9.8 m above horizontal ground.
Take g = 9.8 m s^{-2}.
Find the time before it hits the ground.

Taking up as positive: $-9.8 = 4.9\,t - 4.9t^2$
$4.9t^2 - 4.9\,t - 9.8 = 0$
$t = (4.9 \pm \sqrt{216.09}) \div 9.8 = (4.9 \pm 14.7) \div 9.8$
Positive root only.
Answer: 2 s

8 Perpendicular lines and shortest distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line l has equation $y = -3x + 2$. The point P is $(-7, -7)$.
(a) Find the line through P perpendicular to l, as $ax + by + c = 0$ with integer a, b, c.
(b) Find the foot F and the exact distance PF.

(a) Gradient of l = −3, so the perpendicular has gradient $\frac{1}{3}$

$y + 7 = \frac{1}{3}(x + 7)$, giving $x - 3y - 14 = 0$

(b) Solve $y = -3x + 2$ with $x - 3y - 14 = 0$: F = (2, −4)
$PF^2 = (-9)^2 + (-3)^2 = 90$

Answer: (a) $x - 3y - 14 = 0$ (b) F(2, −4), PF = $3\sqrt{10}$

9 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{5x + 18}{(x + 3)(x + 4)}$ in partial fractions.

(b) Hence evaluate $\int_0^3 \frac{5x + 18}{(x + 3)(x + 4)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{5x + 18}{(x + 3)(x + 4)} \equiv \frac{A}{x + 3} + \frac{B}{x + 4}$, so $5x + 18 \equiv A(x + 4) + B(x + 3)$
$x = -3$: A = 3; $x = -4$: B = 2

(b) $\int_0^3 \left(\frac{3}{x + 3} + \frac{2}{x + 4}\right) dx = [3\ln(x + 3) + 2\ln(x + 4)]_0^3$
$= 3\ln\frac{6}{3} + 2\ln\frac{7}{4} = \ln\left(\frac{49}{2}\right)$

Answer: (a) A = 3, B = 2 (b) $\ln\left(\frac{49}{2}\right)$

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10 Continuous random variable: median (CIE)

The continuous random variable X has pdf $f(x) = \frac{2x}{64}$ for $0 \leq x \leq 8$, and 0 otherwise.

(a) Find P(X > 5).

(b) Find the median of X, to 3 significant figures.

$F(x) = \int_0^x f(t)\,dt = \frac{x^2}{64}$

(a) $P(X > 5) = 1 - F(5) = \frac{39}{64}$

(b) $F(m) = 0.5$: $\frac{m^2}{64} = \frac{1}{2}$

$m = 5.66$

Answer: (a) $\frac{39}{64}$ (b) 5.66