Day 268 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Impulse of a constant force: v = u + at with forces (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A puck of mass 2 kg is moving on a smooth horizontal surface at 9 m s^{-1} when a constant horizontal force of 4 N, in the direction of motion, acts for 3 s.
Find the speed of the puck at the end of this time and the distance it moves.

F = ma: a = 4 ÷ 2 = 2
v = u + at = 9 + 2 × 3
$s = ut + \frac{1}{2}at^2$
Answer: 15 m s^{-1}; 36 m

2 Interpreting regression (AQA, Edexcel, OCR, MEI, Eduqas)

Regression line (x from 2 to 10): y = 3.2 + 0.8x.
x is the advertising spend, y the weekly sales.
(a) Interpret the gradient.
(b) Estimate y when x = 9 and comment.

(a) The gradient: for each extra £100 of advertising, y is predicted to increase by 0.8
(b) y = 3.2 + (0.8) × 9 = 10.4
x = 9 lies within the range of the data (interpolation), so the estimate is reliable
Answer: (a) y increases by 0.8 per extra £100 of advertising (b) 10.4, reliable (interpolation)

3 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Statement: "$n^2 - n + 17$ is prime for every positive integer n."
Find a counter-example to show that the statement is false.

Test n = 1, 2, 3, ... in turn: all give primes until n = 17.
$n^2 - n + 17 = 289 - 17 + 17 = 289$
289 = 17 × 17, which is not prime.
Answer: n = 17 (value 289)

4 Equilibrium of three forces (components) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is in equilibrium under three forces: P N at angle θ above the positive x-axis, a force of 15 N in the negative x-direction and a force of Q N in the negative y-direction.

$\cos\theta = \frac{5}{13}$. Find P and Q.

Resolve horizontally: $P\cos\theta = 15$, so $P \times \frac{5}{13} = 15$

P = 39

Resolve vertically: $Q = P\sin\theta = 39 \times \frac{12}{13}$

Answer: P = 39 N; Q = 36 N

5 Binomial one-tailed test (All boards)

A seed supplier says that 0.3 of seeds germinate. In a sample of 20 seeds, 4 germinate.
Test, at the 10% level, whether the true proportion is lower than claimed.

H_0: p = 0.3, H_1: p < 0.3. Let X ~ B(20, 0.3) under H_0
$P(X \leq 4) = 0.2375$
0.2375 > 0.1, so do not reject H_0
There is insufficient evidence that the proportion is lower than claimed
Answer: do not reject H_0: insufficient evidence that the proportion is lower

6 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $\sum_{r=1}^{7} 2 \times 2^r$.

This is a geometric series with first term 4, common ratio 2 and 7 terms
$S = 4(2^7 - 1) = 508$
Answer: 508

7 Exponential quadratics and ln equations (All boards)

Solve $e^{2x} - 15e^x + 54 = 0$,
giving your answers as exact values.

Let $y = e^x$: $y^2 - 15y + 54 = 0$
$(y - 9)(y - 6) = 0$, so $e^x = 9$ or $e^x = 6$
$x = \ln 6$ or $x = \ln 9$
Answer: $x = \ln 6$, $x = \ln 9$

8 Triangle formed by two lines and an axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Line l_1 has equation $4x - 7y = -4$ and line l_2 has equation $x - y = 2$.
The lines meet at P. Find the area of the triangle formed by l_1, l_2 and the x-axis.

Solve simultaneously: P = (6, 4)
l_1 meets $y = 0$ at $x = -1$, and l_2 meets $y = 0$ at $x = 2$
Base = 3, height = 4
Area = $\frac{1}{2}$ × base × height = 6
Answer: 6

9 R cos(θ − α) form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $9\cos\theta + 12\sin\theta \equiv 15\cos(\theta - 53.1°)$,
solve $9\cos\theta + 12\sin\theta = 5.5$ for $0 \leq \theta \leq 360°$. (1 d.p.)

$15\cos(\theta - 53.1°) = 5.5$, so $\cos(\theta - 53.1°) = \frac{5.5}{15}$
$\theta - 53.1° = \pm 68.5°$
$\theta = 53.1° + 68.5° = 121.6°$
$\theta = 53.1° - 68.5° = -15.4°$, so $\theta = 344.6°$
Answer: θ = 121.6°, 344.6°

10 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{2x + 16}{(x + 2)(x + 5)}$ in partial fractions.

(b) Hence evaluate $\int_0^1 \frac{2x + 16}{(x + 2)(x + 5)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{2x + 16}{(x + 2)(x + 5)} \equiv \frac{A}{x + 2} + \frac{B}{x + 5}$, so $2x + 16 \equiv A(x + 5) + B(x + 2)$
$x = -2$: $A = 4$; $x = -5$: $B = -2$
(b) $\int_0^1 \left(\frac{4}{x + 2} + \frac{-2}{x + 5}\right) dx = [4\ln(x + 2) - 2\ln(x + 5)]_0^1$
$= 4\ln\frac{3}{2} - 2\ln\frac{6}{5} = \ln\left(\frac{225}{64}\right)$
Answer: (a) A = 4, B = −2 (b) $\ln\left(\frac{225}{64}\right)$