**Day 267** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 1 Rough table and a hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 3 kg lies on a rough horizontal table (coefficient of friction 0.2). It is joined by a light string over a smooth pulley to a particle of mass 6 kg hanging freely.

Take g = 9.8 m $s^{-2}$.

The system is released. Find the acceleration and the tension in the string.

Block: $T - 0.2 \times 3\,g = 3\,a$

Hanging: $6\,g - T = 6\,a$

Add: $6\,g - 0.6\,g = 9\,a$

$a = 5.88$

$T = 6(9.8 - 5.88)$

**Answer: a = 5.88 m $s^{-2}$; T = 23.52 N**

## 2 Proof using parity (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $n^2 + 3n + 5$ is odd for all integers n.

$n^2 + 3n = n(n + 3)$

As 3 is odd, n and n + 3 have opposite parity,

so one of them is even and the product n(n + 3) is even.

Even + 5 = odd, so $n^2 + 3n + 5$ is odd.

**Answer: n(n + 3) is even, so the expression is odd**

## 3 Vertical projection upwards (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is projected vertically upwards with speed 19 m $s^{-1}$ from ground level.

Take g = 9.8 m $s^{-2}$. Find, to 3 significant figures,

(a) the greatest height reached, (b) the time of flight.

(a) $0 = 19^2 - 2 \times 9.8 \times H$, so $H = 361 \div 19.6$

(b) s = 0: $t = 2 \times 19 \div 9.8$

**Answer: (a) 18.4 m (b) 3.88 s**

## 4 Quotient rule (All boards)

Find the x-coordinate(s) of the stationary point(s) of the curve $y = \frac{x}{x^2 + 3}$.

$$\frac{dy}{dx} = \frac{3 - x^2}{(x^2 + 3)^2}$$

Stationary points where $\frac{dy}{dx} = 0$, i.e. the numerator is zero

Numerator: $3 - x^2 = 0$

$x = -\sqrt{3}$ or $x = \sqrt{3}$

**Answer: $x = -\sqrt{3}$, $x = \sqrt{3}$**

## 5 Comparing box plots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The box plots show the marks in a mock exam for two groups.

(a) Find the interquartile range of each group.

(b) Compare the two distributions.

(a) Year 12: 50 − 35 = 15. Year 13: 50 − 20 = 30

(b) Medians: 45 (Year 12) and 35 (Year 13), so Year 12 has the higher median

IQRs: Year 13 has the larger IQR, so its scores are more spread out

**Answer: (a) 15 and 30 (b) Year 12 has the higher median; Year 13 is more spread out (larger IQR), Year 12 more consistent**

## 6 Intersection of two lines in 3D (CIE)

Line $l_1$: $\mathbf{r} = \begin{pmatrix} -7 \\ 4 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$.

Line $l_2$: $\mathbf{r} = \begin{pmatrix} -6 \\ 2 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix}$.

Show that $l_1$ and $l_2$ intersect and find the point.

i: $-7 + (-2)\lambda = -6 + 3\mu$

j: $4 + 1\lambda = 2 + 0\mu$

Solving i and j: $\lambda = -2$, $\mu = 1$

k check: $7 + 2(-2) = 3$ and $5 + (-2)(1) = 3$, equal

Point: $\begin{pmatrix} -3 \\ 2 \\ 3 \end{pmatrix}$

**Answer:** $\begin{pmatrix} \mathbf{-3} \\ \mathbf{2} \\ \mathbf{3} \end{pmatrix}$

## 7 Continuous random variable: pdf (CIE)

The continuous random variable X has pdf $f(x) = kx^4$ for $0 \le x \le 4$, and 0 otherwise.

(a) Find the value of k.

(b) Find $P(X > 2)$. (c) Find $E(X)$.

(a) $\int f(x)\,dx = 1$: $k[x^5/5]$ from 0 to 4 $= k \times \frac{4^5}{5} = 1$, so $k = \frac{5}{1024}$

(b) $P(X > 2) = \int_2^4 \frac{5}{1024} x^4\,dx = 1 - \frac{2^5}{4^5} = \frac{31}{32}$

(c) $E(X) = \int_0^4 x\,f(x)\,dx = \frac{5}{1024} \times \frac{4^6}{6} = \frac{10}{3}$

**Answer: (a) $k = \frac{5}{1024}$ (b) $\frac{31}{32}$ (c) $\frac{10}{3}$**

## 8 Exponential quadratics and ln equations (All boards)

Solve $\ln(3x - 1) = 2$, giving your answer to 3 s.f.

$3x - 1 = e^2$

$x = \frac{e^2 + 1}{3} = 2.796$

**Answer: x = 2.80**

## 9 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers m and n such that $15m + 10n = 122$.

Assume there are integers m and n with $15m + 10n = 122$.

$15m + 10n = 5(3m + 2n)$.

So the left-hand side is a multiple of 5.

But $122 = 5 \times 24 + 2$ is not a multiple of 5.

This is a contradiction, so no such integers exist.

**Answer: no integer solutions (LHS is a multiple of 5, 122 is not)**

## 10 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $\sum_{r=1}^{7} 4 \times 2^r$.

This is a geometric series with first term 8, common ratio 2 and 7 terms

$S = 8(2^7 - 1) = 1016$

**Answer: 1016**