Day 266 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Driving force and resistance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lorry of mass 1000 kg moves along a straight horizontal road. The engine produces a driving force of 2000 N and the resistance to motion is 400 N.

Find the acceleration of the lorry.

Resultant force = 2000 − 400 = 1600 N

F = ma: 1600 = 1000 a

Answer: 1.6 m s^{-2}

2 Chain rule (All boards)

Given that $y = e^{4x - 12}$,

find $\frac{dy}{dx}$, and the gradient of the curve when x = 3.

Chain rule: differentiate the outer function, then multiply by the derivative of the inner function

$\frac{dy}{dx} = 4e^{4x - 12}$

When x = 3: gradient = 4

Answer: $\frac{dy}{dx} = 4e^{4x - 12}$; gradient 4

3 Will it move? Limiting friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of mass 12 kg rests on a rough horizontal floor, with coefficient of friction 0.4. A horizontal force of 56 N is applied to the box.

Take g = 9.8 m s^{-2}.

Determine whether the box moves. Find the friction force, or the acceleration if it moves.

R = 12 × 9.8 = 117.6, maximum friction μR = 0.4 × 117.6 = 47.04

56 > 47.04 so the box moves, F = 47.04

56 − 47.04 = 12 a (3 s.f.)

Answer: It moves; a = 0.747 m s^{-2}

4 Parametric curve meeting a line (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations x = −t, y = $-t^2 + t + 3$.

The line y = −2x + 3 meets the curve at two points.

Find the coordinates of these points.

Substitute x and y into the line: $-t^2 + t + 3 = 2t + 3$

$-t^2 - t = 0$, so $-t(t + 1) = 0$

t = −1 or t = 0, giving the points (1, 1) and (0, 3)

Answer: (1, 1) and (0, 3)

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5 Position vectors and dividing a line in a ratio (All boards)

A has position vector $\begin{pmatrix} 0 \\ -3 \\ -3 \end{pmatrix}$ and B has position vector $\begin{pmatrix} 9 \\ -12 \\ -12 \end{pmatrix}$.

The point P lies on AB with AP : PB = 1 : 2.

Find the position vector of P.

$\text{AB} = \begin{pmatrix} 9 \\ -9 \\ -9 \end{pmatrix}$

$\text{AP} = \frac{1}{3}\text{AB} = \begin{pmatrix} 3 \\ -3 \\ -3 \end{pmatrix}$

$\text{OP} = \text{OA} + \text{AP} = \begin{pmatrix} 3 \\ -6 \\ -6 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{3} \\ \mathbf{-6} \\ \mathbf{-6} \end{pmatrix}$

6 Tree diagram (All boards)

The tree diagram shows probabilities for event E, followed by event F.

Find the probability that exactly one of E and F occurs.

P(E and not F) = 0.2 × 0.4 = 0.08

P(not E and F) = 0.8 × 0.1 = 0.08

Exactly one: 0.08 + 0.08 = 0.16

Answer: 0.16

7 Periodic sequences (AQA, Edexcel, OCR, MEI, Eduqas)

A sequence is defined by $u_{n+1} = \frac{1}{1 - u_n}$, $u_1 = -2$.

(a) Show that the sequence is periodic, stating the period.

(b) Find u_{20} and $\sum_{r=1}^{20} u_r$.

(a) $u_2 = \frac{1}{3}$, $u_3 = \frac{3}{2}$, $u_4 = -2$

$u_4 = u_1$ so the sequence repeats with period 3

(b) 20 = 3 × 6 + 2, so $u_{20} = u_2 = \frac{1}{3}$

Sum of one cycle = $-\frac{1}{6}$, so the sum = $6 \times (-\frac{1}{6}) - \frac{5}{3} = -\frac{8}{3}$

Answer: period 3; $u_{20} = \frac{1}{3}$; sum = $-\frac{8}{3}$

8 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix}4\\4\\6\end{pmatrix} + \lambda\begin{pmatrix}0\\-2\\-1\end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix}-2\\-9\\10\end{pmatrix} + \mu\begin{pmatrix}2\\3\\-2\end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $4 + 0\lambda = -2 + 2\mu$

j: $4 + (-2)\lambda = -9 + 3\mu$

Solving i and j: $\lambda = 2$, $\mu = 3$

k check: $6 + (-1)(2) = 4$ and $10 + (-2)(3) = 4$, equal

Point: $\begin{pmatrix}4\\0\\4\end{pmatrix}$

Answer: $\begin{pmatrix}\mathbf{4}\\\mathbf{0}\\\mathbf{4}\end{pmatrix}$

9 Finding mean and standard deviation from two probabilities (All boards)

The masses (kg) of parcels are normally distributed.
$P(X < 10) = 0.0228$ and $P(X > 40) = 0.1587$.
Find the mean and standard deviation (3 s.f.).

$z_1 = -1.999$ and $z_2 = 0.9998$

$10 = \mu + (-1.999)\sigma$ and $40 = \mu + 0.9998\sigma$

Subtract: $30 = 2.999\,\sigma$, so $\sigma = 10.0$

$\mu = 10 - (-1.999)(10.00) = 30.0$

Answer: $\boldsymbol{\mu = 30.0}$, $\boldsymbol{\sigma = 10.0}$

10 Equation of a transformed curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $y = x^2 + x - 3$.
C is stretched by scale factor 3 parallel to the y-axis.
Find its new equation.

Stretch parallel to the y-axis: multiply y by the scale factor.

When stretched by scale factor 3 parallel to the y-axis: $y = 3x^2 + 3x - 9$

Answer: $\boldsymbol{y = 3x^2 + 3x - 9}$