Day 265 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Constant acceleration with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle has velocity $(2\mathbf{i} - 4\mathbf{j})$ m s^{-1} at time t = 0 and moves with constant acceleration $(2\mathbf{i} - \mathbf{j})$ m s^{-2}. Find its velocity and speed when t = 5.

$\mathbf{v} = \mathbf{u} + \mathbf{at} = (2\mathbf{i} - 4\mathbf{j}) + 5(2\mathbf{i} - \mathbf{j})$

$= 12\mathbf{i} - 9\mathbf{j}$

Speed $= \sqrt{(12^2 + -9^2)}$

Answer: $\mathbf{v} = (12\mathbf{i} - 9\mathbf{j})$ m s^{-1}; speed 15 m s^{-1}

2 Chain rule (All boards)

Given that $y = \frac{1}{(3x^2 + 1)}$,

find $\frac{dy}{dx}$, and the gradient of the curve when x = 0.

Chain rule: differentiate the outer function, then multiply by the derivative of the inner function

$\frac{dy}{dx} = -\frac{6x}{(3x^2 + 1)^2}$

When x = 0: gradient = 0

Answer: $\frac{dy}{dx} = -\frac{6x}{(3x^2 + 1)^2}$; gradient 0

3 Inverse functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \frac{-2x - 4}{2x - 2}$ for $x \in \mathbb{R}$, $x \neq 1$.

(a) Find $f^{-1}(x)$.

(b) State the domain of f^{-1}.

Let $y = \frac{-2x - 4}{2x - 2}$. Then $y(2x - 2) = -2x - 4$

$2xy - 2y = -2x - 4$, so $x(2y + 2) = 2y - 4$

$f^{-1}(x) = \frac{2x - 4}{2x + 2}$

f never takes the value -1, so the domain of f^{-1} is $x \neq -1$.

Answer: (a) $f^{-1}(x) = \frac{2x - 4}{2x + 2}$ (b) $x \neq -1$

4 Inverse trigonometric functions (AQA, Edexcel, OCR, MEI, Eduqas)

Find the exact value of $\arcsin(\sin -\frac{\pi}{3})$.

$\sin -\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

$\arcsin(-\frac{\sqrt{3}}{2})$ must lie in $[-\frac{\pi}{2}, \frac{\pi}{2}]$, so the value is $-\frac{\pi}{3}$

Answer: $-\frac{\pi}{3}$

5 Reciprocal graph: asymptotes and intercepts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $y = \frac{-1}{x + 1} - 2$.

(a) Write down the asymptotes.

(b) Find where the curve crosses the axes.

(a) The denominator is zero when $x = -1$, and $y \to -2$ as $x \to \pm\infty$

Asymptotes: $x = -1$ and $y = -2$

(b) x = 0: $y = -1 - 2 = -3$

y = 0: $\frac{-1}{x + 1} = 2$, so $x + 1 = -\frac{1}{2}$, $x = -\frac{3}{2}$

Answer: (a) $x = -1$, $y = -2$ (b) $(0, -3)$ and $(-\frac{3}{2}, 0)$

6 Integration by parts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use integration by parts to find the exact value of

$\int_1^e x \ln x \, dx$.

Let $u = \ln x$ and $\frac{dv}{dx} = x$, so $\frac{du}{dx} = \frac{1}{x}$ and $v = \frac{x^2}{2}$

Integral $= [uv] - \int v\frac{du}{dx} \, dx$

$= [\frac{x^2 \ln x}{2}]_1^e - \int_1^e \frac{x}{2} \, dx$

$= \frac{1}{4} + \frac{e^2}{4}$

Answer: $\mathbf{\frac{1}{4} + \frac{e^2}{4}}$

7 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{20 + 3\sqrt{5}}{\sqrt{5}}$ in the form $p + q\sqrt{5}$, where p and q are integers.

Multiply top and bottom by $\sqrt{5}$:

$\frac{(20 + 3\sqrt{5})\sqrt{5}}{5} = \frac{20\sqrt{5} + 15}{5}$

$= 3 + 4\sqrt{5}$

Answer: $\mathbf{3 + 4\sqrt{5}}$

8 Binomial two-tailed test (All boards)

A factory says a proportion 0.5 of its parts are defective. In a sample of 20 parts, 6 are defective.
Test, at the 5% significance level, whether the proportion is different from 0.5.

H_0: $p = 0.5$, H_1: $p \neq 0.5$. Under H_0, $X \sim B(20, 0.5)$

$P(X \leq 6) = 0.0577$

Two-tailed: compare with 0.025. $0.0577 > 0.025$, so do not reject H_0

There is insufficient evidence that the proportion is different from 0.5

Answer: do not reject H_0: insufficient evidence that the proportion is different from 0.5

9 Particle supported by two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 6 kg is suspended in equilibrium by two light strings attached to a horizontal ceiling. The strings make angles α and β with the ceiling, where $\tan\alpha = \frac{12}{5}$ and $\tan\beta = \frac{4}{3}$.

Take $g = 9.8$ m s^{-2}.
Find the tension in each string.

Horizontal: $T_1 \cos\alpha = T_2 \cos\beta$, $T_1 \times \frac{5}{13} = T_2 \times \frac{3}{5}$

Vertical: $T_1 \sin\alpha + T_2 \sin\beta = 58.8$

From the first: $T_1 = \frac{39}{25} T_2$; into the second: $T_2 \times \frac{56}{25} = 58.8$

Answer: $T_1 = 40.95$ N (at α); $T_2 = 26.25$ N (at β)

10 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Bottle volumes are normal, s.d. 8 ml. Set mean 200 ml; 40 bottles have mean 202.5 ml.
Test at the 5% level whether the true mean is greater than 200.

H_0: $\mu = 200$, H_1: $\mu > 200$. Under H_0, $\overline{X} \sim N(200, \frac{8^2}{40})$

$z = \frac{202.5 - 200}{8/\sqrt{40}} = 1.976$

$P(Z > 1.976) = 0.0241 < 0.05$

Reject H_0: there is sufficient evidence that the mean has increased

Answer: reject H_0: sufficient evidence of an increase in the mean