Day 264 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $\frac{(2x)^2 \times 4x^2}{8x^3}$.

$(2x)^2 = 4x^2$

Numerator $= 4 \times 4x^4 = 16x^4$

Divide: $\frac{16}{8} = 2$ and $x^4 \div x^3 = x$

Answer: $2x$

2 3D vectors: magnitude, distance and unit vectors (All boards)

The vector $\mathbf{a} = \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix}$.

Find $|\mathbf{a}|$ and a unit vector in the direction of $\mathbf{a}$.

$|\mathbf{a}| = \sqrt{3^2 + 2^2 + 6^2} = \sqrt{49} = 7$

Unit vector $= \frac{1}{7}\mathbf{a}$

Answer: $|\mathbf{a}| = 7$, unit vector $\frac{1}{7}\begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix}$

3 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{10 - 5\sqrt{5}}{\sqrt{5}}$ in the form $p + q\sqrt{5}$, where p and q are integers.

Multiply top and bottom by $\sqrt{5}$:

$\frac{(10 - 5\sqrt{5})\sqrt{5}}{5} = \frac{10\sqrt{5} - 25}{5}$

$= -5 + 2\sqrt{5}$

Answer: $-5 + 2\sqrt{5}$

4 Expectation and variance of aX + b (AQA, OCR, MEI, Eduqas, CIE)

The random variable X has $E(X) = 10$ and $\text{Var}(X) = 4$.

Find (a) $E(5X + 1)$,

(b) $\text{Var}(5X + 1)$, (c) $E(X^2)$.

(a) $E(aX + b) = aE(X) + b = 5 \times 10 + 1 = 51$

(b) $\text{Var}(aX + b) = a^2\text{Var}(X) = 25 \times 4 = 100$

(c) $\text{Var}(X) = E(X^2) - [E(X)]^2$, so $E(X^2) = 4 + 100 = 104$

Answer: (a) 51 (b) 100 (c) 104

5 Particle in equilibrium: string and horizontal force (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 6 kg hangs in equilibrium on a light string inclined at α to the vertical, held by a horizontal force P N, as shown. $\tan\alpha = \frac{5}{12}$.

Take $g = 9.8$ m s^{-2}.

Find the tension in the string and the value of P.

$\cos\alpha = 12/13$, $\sin\alpha = 5/13$

Vertical: $T\cos\alpha = mg = 58.8$, so $T = 58.8 \times 13/12$

Horizontal: $P = T\sin\alpha = 63.7 \times 5/13$

Answer: $T = 63.7$ N; $P = 24.5$ N

6 Triangle formed by two lines and an axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Line l_1 has equation $4x - y = -20$ and line l_2 has equation $2x + 3y = 4$.

The lines meet at P. Find the area of the triangle formed by l_1, l_2 and the x-axis.

Solve simultaneously: P = $(-4, 4)$

l_1 meets $y = 0$ at $x = -5$, and l_2 meets $y = 0$ at $x = 2$

Base = 7, height = 4

Area = $\frac{1}{2}$ × base × height = 14

Answer: 14

7 Coded data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

9 values of x are coded using $y = \frac{(x - 200)}{10}$.

$\Sigma y = -3$ and $\Sigma y^2 = 191$.

Find the mean and standard deviation of x (3 s.f.).

Mean of $y = -3 \div 9 = -0.3333$

Mean of $x = 200 + 10 \times -0.3333 = 197$

Variance of $y = \frac{191}{9} - (-0.3333)^2 = 21.11$

SD of y = 4.595, so SD of x = 10 × 4.595 = 45.9

Answer: mean 197, standard deviation 45.9

8 Particle supported by two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 6 kg is suspended in equilibrium by two light strings attached to a horizontal ceiling. The strings make angles α and β with the ceiling, where $\tan\alpha = \frac{5}{12}$ and $\tan\beta = \frac{4}{3}$.

Take $g = 9.8$ m s^{-2}.

Find the tension in each string.

Horizontal: $T_1 \cos\alpha = T_2 \cos\beta$, $T_1 \times \frac{12}{13} = T_2 \times \frac{3}{5}$

Vertical: $T_1 \sin\alpha + T_2 \sin\beta = 58.8$

From the first: $T_1 = \frac{13}{20} T_2$; into the second: $T_2 \times \frac{21}{20} = 58.8$

Answer: T_1 = 36.4 N (at α); T_2 = 56 N (at β)

9 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix} -5 \\ -9 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix} -9 \\ 1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $-5 + 1\lambda = -9 + 3\mu$

j: $-9 + 3\lambda = 1 + (-2)\mu$

Solving i and j: $\lambda = 2$, $\mu = 2$

k check: $2 + (-1)(2) = 0$ and $2 + (-1)(2) = 0$, equal

Point: $\begin{pmatrix} -3 \\ -3 \\ 0 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{-3} \\ \mathbf{-3} \\ \mathbf{0} \end{pmatrix}$

Day 264 **A Level Maths: B Tier 10-a-day: Model Solutions**

10 Integrals of the form f′(x)/f(x) (All boards)

Find the exact value of

$\int_0^{\ln 4} \frac{e^x}{e^x + 1}\, dx,$

giving your answer in the form a ln b.

The numerator is a multiple of the derivative of the denominator

$\int \frac{e^x}{e^x + 1}\, dx = \ln(e^x + 1)$

$= [\ln(e^x + 1)]_0^{\ln 4}$

$= \ln(\frac{5}{2})$

Answer: $\ln(\frac{5}{2})$