Day 263 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Quotient rule (All boards)

Given that $y = \frac{e^{2x}}{x + 3}$,

find $\frac{dy}{dx}$, and its value when $x = 0$.

Quotient rule with $u = e^{2x}$, $v = x + 3$

$u' = 2e^{2x}$, $v' = 1$

$\frac{dy}{dx} = \frac{u'v - uv'}{v^2} = \frac{(2x + 5)e^{2x}}{(x + 3)^2}$

When $x = 0$: $\frac{dy}{dx} = \frac{5}{9}$

Answer: $\frac{dy}{dx} = \frac{(2x + 5)e^{2x}}{(x + 3)^2}$**; value** $\frac{5}{9}$

2 Resultant of perpendicular forces (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Two forces of magnitude 9 N and 12 N act at right angles on a particle.
Find the magnitude of the resultant, and the angle it makes with the 9 N force, to 1 d.p.

Magnitude $= \sqrt{(9^2 + 12^2)} = \sqrt{225}$

$\tan \alpha = 12 \div 9$

Answer: 15 N at 53.1°

3 Exact values of sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\tan \theta = \frac{1}{2}$ and θ is acute,

find the exact value of $\sec \theta$.

Use $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$\tan \theta = \frac{1}{2}$, so $\sec^2\theta = 1 + \frac{1}{4} = \frac{5}{4}$

$\sec \theta = \frac{\sqrt{5}}{2}$ (positive as θ is acute)

Answer: $\frac{\sqrt{5}}{2}$

4 Factor theorem with an unknown (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 2x^3 + kx^2 + 8x$.
Given that $(x + 4)$ is a factor of $f(x)$,
(a) find the value of k,
(b) factorise $f(x)$ completely.

$f(-4) = 0$: $-128 + 16k - 32 = 0$

$k = 10$

$f(x) = (x + 4)(2x^2 + 2x)$

$= 2x(x + 1)(x + 4)$

Answer: (a) k = 10 (b) f(x) = 2x(x + 1)(x + 4)

5 Impulse of a constant force: v = u + at with forces (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 8 kg is moving on a smooth horizontal surface at 1 m s^{-1} when a constant horizontal force of 12 N, in the direction of motion, acts for 3 s.
Find the speed of the particle at the end of this time and the distance it moves.

$F = ma$: $a = 12 \div 8 = 1.5$

$v = u + at = 1 + 1.5 \times 3$

$s = ut + \frac{1}{2}at^2$

Answer: 5.5 m s^{-1}; 9.75 m

6 Finding mean and standard deviation from two probabilities (All boards)

The masses (kg) of parcels are normally distributed.
P(X < 90) = 0.1587 and P(X > 110) = 0.1587.
Find the mean and standard deviation (3 s.f.).

$z_1 = -0.9998$ and $z_2 = 0.9998$
$90 = \mu + (-0.9998)\sigma$ and $110 = \mu + 0.9998\sigma$
Subtract: $20 = 2.000\,\sigma$, so $\sigma = 10.0$
$\mu = 90 - (-0.9998)(10.00) = 100$
Answer: $\mu = 100$, $\sigma = 10.0$

7 Vectors in a triangle (proof and ratio) (All boards)

In triangle OAB, OA = a and OB = b. M lies on AB with AM : MB = 1 : 3, and N lies on OB with ON : NB = 1 : 2.
Find OM and NM in terms of a and b.

$AB = b - a$, so $AM = \frac{1}{4}(b - a)$

$OM = a + AM = \frac{3a}{4} + \frac{b}{4}$

$ON = \frac{b}{3}$, so $NM = OM - ON = \frac{3a}{4} - \frac{b}{12}$

Answer: OM = $\frac{3a}{4} + \frac{b}{4}$, NM = $\frac{3a}{4} - \frac{b}{12}$

8 Grouped data: mean and median (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table summarises 50 measurements of t.
(a) Estimate the mean.
(b) Estimate the median using linear interpolation.

(a) Using mid-points: $\Sigma fx = 3340$, so mean = $3340 \div 50 = 66.8$

(b) $\frac{n}{2} = 25$. Cumulative frequency before the median class = 20

Median class 60 to 70. Median = $60 + \frac{5}{7} \times 10 = 67.1$

Answer: mean ≈ 66.8, median ≈ 67.1

9 Coefficient in a product with a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^2 in the expansion of
$(2 + 4x)(3 + x)^4$.

$(3 + x)^4$ has x^1 coefficient 108 and x^2 coefficient 54
x^2 term of the product = $2 \times 54 + 4 \times 108$
$= 540$
Answer: 540

10 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{4}{1 - \sqrt{2}}$ in the form $p + q\sqrt{2}$, where p and q are integers.

Multiply top and bottom by the conjugate $(1 + \sqrt{2})$:
Denominator: $(1 - \sqrt{2})(1 + \sqrt{2}) = 1^2 - 1^2 \times 2 = -1$
Numerator: $(4)(1 + \sqrt{2}) = 4 + 4\sqrt{2}$
So the fraction = $\frac{4 + 4\sqrt{2}}{-1} = -4 - 4\sqrt{2}$

Answer: $-4 - 4\sqrt{2}$