Day 262 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 3D vectors: magnitude, distance and unit vectors (All boards)

Points A and B have position vectors $\begin{pmatrix}1\\1\\0\end{pmatrix}$ and $\begin{pmatrix}-2\\13\\4\end{pmatrix}$.

Find AB and the distance AB.

$\text{AB} = \text{OB} - \text{OA} = \begin{pmatrix}-3\\12\\4\end{pmatrix}$

$|\text{AB}| = \sqrt{3^2 + 12^2 + 4^2} = \sqrt{169} = 13$

Answer: AB $= \begin{pmatrix}\mathbf{-3}\\\mathbf{12}\\\mathbf{4}\end{pmatrix}$**, distance 13**

2 Equation of a line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A and B have coordinates $(0, -5)$ and $(-2, 1)$.

(a) Find the midpoint of AB.

(b) Find the line AB as $ax + by + c = 0$, with integer a, b, c.

(a) Midpoint $= \left(\frac{0 + (-2)}{2}, \frac{-5 + 1}{2}\right) = (-1, -2)$

(b) Gradient $= \frac{6}{-2} = -3$

$y + 5 = -3x$

Multiply out and collect terms: $3x + y + 5 = 0$

Answer: (a) (−1, −2) (b) 3x + y + 5 = 0

3 Braking distance and time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car travelling at 16 m s^{-1} brakes and comes to rest with constant deceleration 1.6 m s^{-2}.

Find (a) the distance travelled while braking, (b) the time taken to stop.

(a) $0 = 16^2 - 2 \times 1.6 \times s$

$s = 256 \div 3.2 = 80$

(b) $0 = 16 - 1.6\,t$, so $t = 16 \div 1.6$

Answer: (a) 80 m (b) 10 s

4 Frequency table: mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table shows the number of siblings, x, of 32 students.

Calculate the mean and standard deviation (3 s.f.).

$\Sigma f = 32$, $\Sigma fx = 86$, $\Sigma fx^2 = 274$

Mean $= \frac{86}{32} = 2.69$

Variance $= \frac{274}{32} - (2.6875)^2 = 1.340$

Standard deviation = 1.16

Answer: mean 2.69, standard deviation 1.16

5 Constant velocity: position vector (AQA, Edexcel, OCR, MEI, Eduqas)

A drone moves with constant velocity $(2\mathbf{i} - \mathbf{j})$ m s^{-1}. At time t = 0 its position vector relative to a fixed origin O is $(-3\mathbf{i} + 2\mathbf{j})$ m.

Find (a) its position vector when t = 4, (b) the value of t when it crosses the line through O in the direction of $\mathbf{i}$.

(a) $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t = (-3\mathbf{i} + 2\mathbf{j}) + 4(2\mathbf{i} - \mathbf{j}) = (5\mathbf{i} - 2\mathbf{j})$

(b) $\mathbf{j}$ component zero: $2 - t = 0$, so $t = 2$

Answer: (a) (5i − 2j) m (b) t = 2 s

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6 Quadratic equations in sin x, cos x, tan x (All boards)

Solve $12\cos^2 x + 1 - 7\cos x = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p. where necessary.

$(4\cos x - 1)(3\cos x - 1) = 0$, so $\cos x = \frac{1}{4}$ or $\cos x = \frac{1}{3}$

$\cos x = \frac{1}{4}$: $x = 75.5°, 284.5°$

$\cos x = \frac{1}{3}$: $x = 70.5°, 289.5°$

Answer: x = 70.5°, 75.5°, 284.5°, 289.5°

7 Histogram bar heights (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A class of width 10, frequency 36, is a bar 5 cm wide, 1.8 cm tall.
(a) Find the bar height for width 10, frequency 24.
(b) Find the frequency of a 3 cm by 3 cm bar.

Area of bar $\propto$ frequency: $5 \times 1.8 = 9$ cm^2 represents 36
so 1 unit of frequency = 0.25 cm^2
(a) Width = 5 cm, area = $0.25 \times 24 = 6$ cm^2, height = $6 \div 5 = 1.2$ cm
(b) Area = $3 \times 3 = 9$ cm^2, frequency = $9 \div 0.25 = 36$
Answer: (a) 1.2 cm (b) 36

8 Position vectors and dividing a line in a ratio (All boards)

A has position vector $\begin{pmatrix}2\\1\\5\end{pmatrix}$ and B has position vector $\begin{pmatrix}-3\\16\\-5\end{pmatrix}$.

The point P lies on AB with AP : PB = 2 : 3.
Find the position vector of P.

$\text{AB} = \begin{pmatrix}-5\\15\\-10\end{pmatrix}$

$\text{AP} = \frac{2}{5}\text{AB} = \begin{pmatrix}-2\\6\\-4\end{pmatrix}$

$\text{OP} = \text{OA} + \text{AP} = \begin{pmatrix}0\\7\\1\end{pmatrix}$

Answer: $\begin{pmatrix}\mathbf{0}\\\mathbf{7}\\\mathbf{1}\end{pmatrix}$

9 Transformations from a graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x). A and B are the turning points. Find the turning points of:
(a) y = f(x + 3) (b) y = f(x) + 4

(a) y = f(x + 3): A(−2, −3) maps to (−5, −3), B(4, 2) maps to (1, 2)
(b) y = f(x) + 4: A(−2, −3) maps to (−2, 1), B(4, 2) maps to (4, 6)
Answer: (a) (−5, −3), (1, 2) (b) (−2, 1), (4, 6)

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10 Partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{x^2 - 15x - 61}{(x + 1)(x + 4)^2}$ in partial fractions.

$\frac{x^2 - 15x - 61}{(x + 1)(x + 4)^2} \equiv \frac{A}{(x + 1)} + \frac{B}{(x + 4)} + \frac{C}{(x + 4)^2}$

$x^2 - 15x - 61 \equiv A(x + 4)^2 + B(x + 1)(x + 4) + C(x + 1)$

$x = -1$ gives $A = -5$; $x = -4$ gives $C = -5$

Coefficients of x^2: $1 = A + B$, so $B = 6$

Answer: $\mathbf{-\frac{5}{(x + 1)} + \frac{6}{(x + 4)} - \frac{5}{(x + 4)^2}}$