Day 261 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Union, conditional probability and independence (All boards)

P(A) = 0.25, P(B) = 0.6, P(A ∩ B) = 0.09.
Find (a) P(A ∪ B), (b) P(A | B).
(c) Are A and B independent?

(a) $P(A \cup B) = 0.25 + 0.6 - 0.09 = 0.76$

(b) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.09}{0.6} = 0.15$

(c) $P(A)P(B) = 0.25 \times 0.6 = 0.15$, which does not equal $P(A \cap B)$

Answer: (a) 0.76 (b) 0.15 (c) not independent

2 Proof by deduction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $(n + 2)^2 - (n - 2)^2$ is a multiple of 8 for all integer values of n.

$(n + 2)^2 = n^2 + 4n + 4$

$(n - 2)^2 = n^2 - 4n + 4$

Difference $= 8n = 8 \times n$

n is an integer, so this is a multiple of 8 for every integer n.

Answer: difference = 8n, a multiple of 8

3 Expectation and variance of aX + b (AQA, OCR, MEI, Eduqas, CIE)

The random variable X has E(X) = 6 and Var(X) = 9.
Find (a) E(3X + 3),
(b) Var(3X + 3), (c) $E(X^2)$.

(a) $E(aX + b) = aE(X) + b = 3 \times 6 + 3 = 21$

(b) $Var(aX + b) = a^2Var(X) = 9 \times 9 = 81$

(c) $Var(X) = E(X^2) - [E(X)]^2$, so $E(X^2) = 9 + 36 = 45$

Answer: (a) 21 (b) 81 (c) 45

4 Vectors in a triangle (proof and ratio) (All boards)

In triangle OAB, OA = a and OB = b. M lies on AB with AM : MB = 3 : 1, and N lies on OB with ON : NB = 3 : 1.
Find OM and NM in terms of a and b.

$AB = b - a$, so $AM = \frac{3}{4}(b - a)$

$OM = a + AM = \frac{a}{4} + \frac{3b}{4}$

$ON = \frac{3b}{4}$, so $NM = OM - ON = \frac{a}{4}$

Answer: OM = $\frac{a}{4} + \frac{3b}{4}$, NM = $\frac{a}{4}$

5 Power, driving force and resistance (CIE)

A car of mass 1000 kg moves along a horizontal road. The engine works at a constant rate of 24 kW. The resistance to motion is constant at 600 N.
Find the acceleration when the speed is 10 m s^{-1}.

$P = Fv$: $F = 24000 \div 10 = 2400$ N

$F - R = ma$: $2400 - 600 = 1000a$

Answer: 1.8 m s^{-2}

6 Inverse functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \sqrt{x + 4}$ for $x \geq -4$.
Find $f^{-1}(x)$ and state its domain.

Let $y = \sqrt{x + 4}$. Then $y^2 = x + 4$

$x = y^2 - 4$

f(x) ≥ 0, so the domain of f^{-1} is $x \geq 0$.

Answer: $f^{-1}(x) = x^2 - 4$, domain $x \geq 0$

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7 Product rule (All boards)

Given that $y = x^3 \cos 3x$,

find $\frac{dy}{dx}$, and its value when $x = \frac{\pi}{3}$.

Product rule with $u = x^3$, $v = \cos 3x$

$u' = 3x^2$, $v' = -3 \sin 3x$

$\frac{dy}{dx} = u'v + uv' = 3x^2(\cos 3x - x \sin 3x)$

When $x = \frac{\pi}{3}$: $\frac{dy}{dx} = -\frac{\pi^2}{3}$

Answer: $\frac{dy}{dx} = 3x^2(\cos 3x - x \sin 3x)$; value $-\frac{\pi^2}{3}$

8 Tangent lines of a given gradient (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $3x - 4y = k$ is a tangent to the circle

$(x - 2)^2 + y^2 = 100$.

Find the two possible values of k.

The radius to the point of contact is perpendicular to the line, so it is parallel to $(3, -4)$.
A unit step in this direction is $(3, -4)/5$, so the points of contact are
$(2, 0) \pm 10 \times (3, -4)/5 = (2, 0) \pm (6, -8)$
T = $(8, -8)$ or $(-4, 8)$, and $k = 3x - 4y$ gives:
k = 56 or k = −44
Answer: k = 56 or k = −44

9 R cos(θ − α) form: express and maximum (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\cos\theta + \sin\theta$ in the form $R\cos(\theta - \alpha)$, with $R > 0$ and $0 < \alpha < 90°$.
(b) Hence find its maximum value and the smallest $\theta \geq 0$ at which it occurs. Give α to 1 d.p.

$R = \sqrt{1^2 + 1^2} = \sqrt{2}$, $\tan\alpha = \frac{1}{1}$ so $\alpha = 45.0°$

Maximum is R when $\cos(\theta - \alpha) = 1$, i.e. $\theta = \alpha$

Answer: (a) $\sqrt{2}\cos(\theta - 45.0°)$ (b) max $\sqrt{2}$ at $\theta = 45.0°$

10 Differentiating position vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves so that its position vector relative to O at time t s is $\mathbf{r} = (3t^2 - t)\mathbf{i} + (3t^3 + 3t)\mathbf{j}$ metres.
Find the velocity when t = 1, and the speed at that time.

$\mathbf{v} = d\mathbf{r}/dt = (6t - 1)\mathbf{i} + (9t^2 + 3)\mathbf{j}$
t = 1: $\mathbf{v} = (5)\mathbf{i} + (12)\mathbf{j}$
Speed = $\sqrt{(5^2 + 12^2)}$
Answer: $\mathbf{v} = (5)\mathbf{i} + (12)\mathbf{j}$ m s^{-1}; speed 13 m s^{-1}