Day 260 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 3D vectors: magnitude, distance and unit vectors (All boards)

Points A and B have position vectors $\begin{pmatrix}-4\\2\\1\end{pmatrix}$ and $\begin{pmatrix}-16\\-10\\2\end{pmatrix}$.

Find AB and the distance AB.

$\text{AB} = \text{OB} - \text{OA} = \begin{pmatrix}-12\\-12\\1\end{pmatrix}$

$|\text{AB}| = \sqrt{12^2 + 12^2 + 1^2} = \sqrt{289} = 17$

Answer: AB $= \begin{pmatrix}\mathbf{-12}\\\mathbf{-12}\\\mathbf{1}\end{pmatrix}$**, distance 17**

2 Equation of a circle from given information (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has centre $(-4, -1)$ and passes through the point $(5, -4)$.

Find an equation of the circle.

$r^2 = (5 - (-4))^2 + (-4 - (-1))^2$

$= 81 + 9 = 90$

Answer: $\mathbf{(x + 4)^2 + (y + 1)^2 = 90}$

3 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

10 pairs of rainfall and crop yield give $r = 0.462$.

One-tailed critical value (5%): 0.5494.

Test for positive correlation.

$H_0: \rho = 0$, $H_1: \rho > 0$

$r = 0.462$, critical value 0.5494; r is not beyond the critical value in the right direction

r is not in the critical region, so do not reject H_0

Conclusion: there is insufficient evidence of positive correlation between rainfall and crop yield

Answer: do not reject $\mathbf{H_0}$**: there is insufficient evidence of positive correlation between rainfall and crop yield**

4 Exponential growth and decay models (All boards)

A car's value, £V, after t years is given by $V = 12000 \times 0.92^t$.

Find (a) the value after 3 years, (b) when it first falls below £3000, to 3 s.f.

(a) $V = 12000 \times 0.92^3 = 9344.3$

(b) $0.92^t < 0.25$, so $t \ln 0.92 < \ln 0.25$

$\ln 0.92 < 0$, so the inequality reverses: $t > \dfrac{\ln 0.25}{\ln 0.92} = 16.63$

Answer: (a) £9344 (b) $\mathbf{t > 16.6}$ **years**

5 Inverse functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 3x^2 + 18x + 31$ for $x \geq -3$.

Find $f^{-1}(x)$ and state its domain.

Complete the square: $f(x) = 3(x + 3)^2 + 4$

$y = 3(x + 3)^2 + 4$, so $(x + 3)^2 = \dfrac{y - 4}{3}$

As $x + 3 \geq 0$ take the positive root: $x = -3 + \sqrt{\dfrac{y - 4}{3}}$

As $x + 3 \geq 0$, take the positive root: $x = -3 + \sqrt{\dfrac{y - 4}{3}}$

Answer: $\mathbf{f^{-1}(x) = -3 + \sqrt{\dfrac{x - 4}{3}}}$**, domain** $\mathbf{x \geq 4}$

6 Projectile at an angle: time and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is projected from level ground with horizontal velocity component 26 m s^{-1} and vertical velocity component 9.8 m s^{-1} upwards.

Take g = 9.8 m s^{-2}.

Find (a) the time of flight, (b) the horizontal range.

(a) Vertical: $0 = 9.8\,t - 4.9t^2$, so $t = 9.8 \div 4.9$

(b) Range = horizontal speed × time = 26 × 2

Answer: (a) 2 s (b) 52 m

7 Finding the mean or standard deviation (All boards)

A battery lasts X hours, normal with mean 75.

Given P(X < 84.3) = 0.99, find the standard deviation (3 s.f.).

$z = 2.326$, so $\frac{84.3 - 75}{\sigma} = 2.326$

$\sigma = 9.3 \div 2.326 = 4.00$

Answer: σ = 4.00

8 Equations with sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $3\tan^2 x + 10\sec x + 11 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p.

Use $\tan^2 x = \sec^2 x - 1$: $3\sec^2 x + 10\sec x + 8 = 0$

$(\sec x + 2)(3\sec x + 4) = 0$, so $\sec x = -2$ or $-\frac{4}{3}$

$\sec x = -2$, so $\cos x = -\frac{1}{2}$: $x = 120.0°, 240.0°$

$\sec x = -\frac{4}{3}$, so $\cos x = -\frac{3}{4}$: $x = 138.6°, 221.4°$

Answer: x = 120.0°, 138.6°, 221.4°, 240.0°

9 Partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{11x - 26}{(2x - 5)(x - 4)}$ in partial fractions.

$\frac{11x - 26}{(2x - 5)(x - 4)} \equiv \frac{A}{(2x - 5)} + \frac{B}{(x - 4)}$, so $11x - 26 \equiv A(x - 4) + B(2x - 5)$

$x = \frac{5}{2}$: $\frac{3}{2} = A \times (-\frac{3}{2})$, so $A = -1$

$x = 4$: $18 = B \times 3$, so $B = 6$

Answer: $-\frac{1}{(2x - 5)} + \frac{6}{(x - 4)}$

10 Projectile: greatest height and speed at the top (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A rocket is projected from ground level with initial velocity components 17 m s^{-1} horizontally and 39.2 m s^{-1} vertically upwards.

Take g = 9.8 m s^{-2}.

Find (a) the greatest height, (b) the speed at the greatest height.

(a) $0 = 39.2^2 - 2 \times 9.8 \times H$, $H = 1536.64 \div 19.6$

(b) At the top the vertical velocity is zero, so the speed is the horizontal component.

Answer: (a) 78.4 m (b) 17 m s^{-1}