Day 26 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Product moment correlation coefficient (AQA, Edexcel, OCR, MEI, Eduqas)

For 15 pairs of data, $S_{xx} = 72$, $S_{yy} = 60$ and $S_{xy} = 53$.

Find the PMCC (3 s.f.) and describe the correlation.

$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{53}{\sqrt{72 \times 60}}$

$= \frac{53}{65.727} = 0.806$

r is positive, so there is strong positive correlation

Answer: r = 0.806, strong positive correlation

2 Velocity parallel to a direction (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves so that at time t s its velocity is $\mathbf{v} = (3t^2 + 4)\mathbf{i} + (6t - 12)\mathbf{j}$ m s^{-1}.

Find the time when the particle is moving in the direction of $\mathbf{i}$, and its speed then.

Parallel to $\mathbf{i}$: $\mathbf{j}$ component = 0

$6t - 12 = 0$, so $t = 2$

$\mathbf{v} = (3 \times 2^2 + 4)\mathbf{i} = 16\mathbf{i}$

Answer: t = 2 s, speed 16 m s^{-1}

3 Standard integrals (definite) (All boards)

Find the exact value of

$\int_1^2 (5x^3 + 1 - \frac{4}{x^2})\,dx$.

$\int (5x^3 + 1 - \frac{4}{x^2})\,dx = \frac{5x^4}{4} + x + \frac{4}{x}$

$= [\frac{5x^4}{4} + x + \frac{4}{x}]_1^2$

$= \frac{71}{4}$

Answer: $\frac{71}{4}$

4 Algebraic division (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the quotient and the remainder when

$2x^3 - 7x^2 + 2x - 18$ is divided by $(x - 4)$.

$2x^2 \times (x - 4) = 2x^3 - 8x^2$, leaving $x^2 + 2x - 18$

$x \times (x - 4) = x^2 - 4x$, leaving $6x - 18$

$6 \times (x - 4) = 6x - 24$, leaving 6

Quotient $2x^2 + x + 6$, remainder 6

Answer: quotient $2x^2 + x + 6$, remainder 6

5 Rod hinged at one end and held by a string (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 2 m and mass 18 kg is hinged at A and held horizontal by a light string attached at B.

The string makes an angle θ with the rod, where $\sin\theta = \frac{4}{5}$. Take g = 9.8 m s^{-2}. Find the tension in the string.

Moments about A: $T \sin\theta \times 2 = mg \times 1$

$T \times \frac{4}{5} \times 2 = 176.4 \times 1$

$T = 176.4 \div 1.6$

Answer: 110.25 N

6 Discriminant problems (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The equation $x^2 + 2kx + (k + 6) = 0$ has no real roots.
Find the possible values of k.

$b^2 - 4ac = (2k)^2 - 4(k + 6) = 4k^2 - 4k - 24$
$= 4(k^2 - k - 6) = 4(k - 3)(k + 2)$
Need $b^2 - 4ac < 0$, so $-2 < k < 3$.
Answer: $-2 < k < 3$

7 Solving exponential equations using logarithms (All boards)

Solve $2^{3x+1} = 159$, giving your answer to 3 s.f.

Take logs: $(3x + 1) \ln 2 = \ln 159$
$3x + 1 = \frac{\ln 159}{\ln 2} = 7.3129$
$x = 2.104$
Answer: $x = 2.10$

8 Geometric series and logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has first term 6 and common ratio 2.
Find the smallest value of n for which the sum of the first n terms exceeds 20000.

$S_n = 6(2^n - 1) > 20000$
$2^n > 3334.33$
$n > \frac{\lg 3334.33}{\lg 2} = 11.703$
So the smallest integer n is 12
Answer: $n = 12$

9 Finding mean and standard deviation from two probabilities (All boards)

The random variable X is normally distributed.
$P(X < 42.5) = 0.0668$ and $P(X > 55) = 0.1587$.
Find the mean and standard deviation (3 s.f.).

$z_1 = -1.500$ and $z_2 = 0.9998$
$42.5 = \mu + (-1.500)\sigma$ and $55 = \mu + 0.9998\sigma$
Subtract: $12.5 = 2.500\ \sigma$, so $\sigma = 5.00$
$\mu = 42.5 - (-1.500)(5.000) = 50.0$
Answer: $\mu = 50.0$, $\sigma = 5.00$

10 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix} -6 \\ -3 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $-6 + 1\lambda = 3 + (-2)\mu$
j: $-3 + 2\lambda = 0 + 1\mu$
Solving i and j: $\lambda = 3$, $\mu = 3$
k check: $6 + (-1)(3) = 3$ and $3 + 0(3) = 3$, equal
Point: $\begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix}$
Answer: $\begin{pmatrix} \mathbf{-3} \\ \mathbf{3} \\ \mathbf{3} \end{pmatrix}$