Day 259 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Mean and standard deviation from sums (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A summary of the ages, a years, of 7 members of a club gives $\Sigma a = 144$ and $\Sigma a^2 = 3612$.
Find the mean and the standard deviation (3 s.f.).

Mean = $144 \div 7 = 20.6$

Variance = $\frac{3612}{7} - (20.571)^2 = 92.82$

Standard deviation = $\sqrt{92.82} = 9.63$

Answer: mean 20.6, standard deviation 9.63

2 Scalar product and angle between vectors (CIE)

$\mathbf{a} = \begin{pmatrix} 0 \\ 2 \\ -4 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 5 \\ 0 \end{pmatrix}$.

Find the angle between a and b, to 1 d.p.

$\mathbf{a}\cdot\mathbf{b} = (0)(-2) + (2)(5) + (-4)(0) = 10$

$|\mathbf{a}| = \sqrt{20}$, $|\mathbf{b}| = \sqrt{29}$

$\cos\theta = \frac{10}{\sqrt{20}\sqrt{29}} = 0.4152$

$\theta = 65.5°$

Answer: 65.5°

3 Newton's second law: lift and tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 400 kg is raised and lowered by a vertical cable. The lift accelerates downwards at 2 m s^{-2}.
Take g = 9.8 m s^{-2}.
Find the tension in the cable.

Resolve vertically (up positive): $T - 400 \times 9.8 = -400 \times 2$

$T = 400(9.8 - 2)$

Answer: 3120 N

4 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $\frac{(4x^3)^3 \times 3x}{6x^3}$.

$(4x^3)^3 = 64x^9$

Numerator = $64 \times 3x^{10} = 192x^{10}$

Divide: $\frac{192}{6} = 32$ and $x^{10} \div x^3 = x^7$

Answer: $32x^7$

5 Area under a curve (All boards)

The diagram shows the curve $y = x(8 - x)$ and the line $y = 3x$.
Find the exact area of the shaded region R.

Intersect: $x(8 - x) = 3x$, so $x = 0$ or $x = 5$

Area = $\int_0^5 (x(8 - x) - 3x)\,dx = \int_0^5 (5x - x^2)\,dx$

$= [\frac{5x^2}{2} - \frac{x^3}{3}]_0^5 = \frac{125}{6}$

Answer: $\frac{125}{6}$

6 Rod suspended by two vertical strings (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 4 m and mass 8 kg hangs horizontally from two vertical strings attached 0.5 m from A and 0.5 m from B.

A particle of mass 5 kg is attached 2 m from A. Take g = 9.8 m s^{-2}. Find the tension in each string.

Moments about the string near A: $T_2(3) = 8\,g(1.5) + 5\,g(1.5)$

$T_2 = 63.7$

$T_1 + T_2 = 13\,g$ so $T_1 = 127.4 - 63.7$

Answer: Near A: 63.7 N; near B: 63.7 N

7 Periodic sequences (AQA, Edexcel, OCR, MEI, Eduqas)

A sequence is defined by $u_{n+1} = \frac{u_n - 1}{u_n + 1}$, $u_1 = 2$.

(a) Show that the sequence is periodic, stating the period.

(b) Find u_{30} and $\sum_{r=1}^{30} u_r$.

(a) $u_2 = \frac{1}{3}$, $u_3 = -\frac{1}{2}$, $u_4 = -3$, $u_5 = 2$

$u_5 = u_1$ so the sequence repeats with period 4

(b) $30 = 4 \times 7 + 2$, so $u_{30} = u_2 = \frac{1}{3}$

Sum of one cycle $= -\frac{7}{6}$, so the sum $= 7 \times (-\frac{7}{6}) + \frac{7}{3} = -\frac{35}{6}$

Answer: period 4; $u_{30} = \frac{1}{3}$; sum $= -\frac{35}{6}$

8 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

8 pairs of hours of sleep and reaction time give $r = -0.819$.

One-tailed critical value (1%): 0.7887.

Test for negative correlation.

$H_0: \rho = 0$, $H_1: \rho < 0$

$r = -0.819$, critical value -0.7887; r is beyond the critical value in the right direction

r is in the critical region, so reject H_0

Conclusion: there is evidence of negative correlation between hours of sleep and reaction time

Answer: reject H_0: there is evidence of negative correlation between hours of sleep and reaction time

9 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix} -1 \\ -3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix} 1 \\ -3 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $-1 + 1\lambda = 1 + 0\mu$

j: $-3 + 1\lambda = -3 + (-1)\mu$

Solving i and j: $\lambda = 2$, $\mu = -2$

k check: $1 + 0(2) = 1$ and $-1 + (-1)(-2) = 1$, equal

Point: $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{1} \\ \mathbf{-1} \\ \mathbf{1} \end{pmatrix}$

Day 259 **A Level Maths: B Tier 10-a-day: Model Solutions**

10 Combined inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x for which
$2x - 6 > -5$ and $x^2 - 6x + 5 \geq 0$.

$2x - 6 > -5$ gives $x > \frac{1}{2}$

$x^2 - 6x + 5 \geq 0$: $(x - 1)(x - 5) \geq 0$ gives $x \leq 1$ or $x \geq 5$

Both must hold: $\frac{1}{2} < x \leq 1$ or $x \geq 5$

Answer: $\frac{1}{2} < x \leq 1$ or $x \geq 5$