Day 258 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A school has 64 students in Year 7, 20 in Year 8 and 36 in Year 9.
A stratified sample of size 30 is taken. Find the number selected from each group.

Total = 64 + 20 + 36 = 120

Sampling fraction = 30/120 = $\frac{1}{4}$

Year 7: 64 ÷ 4 = 16
Year 8: 20 ÷ 4 = 5
Year 9: 36 ÷ 4 = 9

Answer: Year 7 16, Year 8 5, Year 9 9

2 Rough inclined plane: sliding down (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 6 kg is released from rest on a rough plane inclined at angle α to the horizontal, where $\tan\alpha = \frac{5}{12}$.

The coefficient of friction is 0.2. Take g = 9.8 m s^{-2}. Find the acceleration down the plane, and the distance moved in 3 s.

$\cos\alpha = 12/13$, $\sin\alpha = 5/13$
$R = mg\cos\alpha$, $F = \mu R$
$ma = mg\sin\alpha - \mu mg\cos\alpha$, so $a = 9.8(5/13 - 0.2 \times 12/13)$
$s = \frac{1}{2}at^2 = \frac{1}{2} \times 1.96 \times 3^2$

Answer: 1.96 m s^{-2}; 8.82 m

3 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $\frac{(2x)^3 \times x^3}{4x^3}$.

$(2x)^3 = 8x^3$
Numerator = $8x^3 \times x^3 = 8x^6$
Divide: $\frac{8}{4} = 2$ and $x^6 \div x^3 = x^3$

Answer: $2x^3$

4 Differentiating position vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves so that its position vector relative to O at time t s is $\mathbf{r} = (2t^2 + t)\mathbf{i} + (3t^3 + 3t)\mathbf{j}$ metres.
Find the velocity when t = 1, and the speed at that time.

$\mathbf{v} = d\mathbf{r}/dt = (4t + 1)\mathbf{i} + (9t^2 + 3)\mathbf{j}$
t = 1: $\mathbf{v} = (5)\mathbf{i} + (12)\mathbf{j}$
Speed = $\sqrt{(5^2 + 12^2)}$

Answer: $\mathbf{v} = (5)\mathbf{i} + (12)\mathbf{j}$ m s^{-1}; speed 13 m s^{-1}

5 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{-3 + 4\sqrt{3}}{2 - \sqrt{3}}$ in the form $p + q\sqrt{3}$, where p and q are integers.

Multiply top and bottom by the conjugate $(2 + \sqrt{3})$:
Denominator: $(2 - \sqrt{3})(2 + \sqrt{3}) = 2^2 - 1^2 \times 3 = 1$
Numerator: $(-3 + 4\sqrt{3})(2 + \sqrt{3}) = 6 + 5\sqrt{3}$
So the fraction = $\frac{6 + 5\sqrt{3}}{1} = 6 + 5\sqrt{3}$

Answer: $6 + 5\sqrt{3}$

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6 Mean and standard deviation from sums (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A summary of the times, t minutes, taken by 10 people to complete a puzzle gives $\Sigma t = 277$ and $\Sigma t^2 = 8881$.
Find the mean and the standard deviation (3 s.f.).

Mean = $277 \div 10 = 27.7$

Variance = $\frac{8881}{10} - (27.700)^2 = 120.8$

Standard deviation = $\sqrt{120.8} = 11.0$

Answer: mean 27.7, standard deviation 11.0

7 Simplify using double angle formulae (All boards)

Simplify $\frac{1 - \cos 6x}{2 \sin 3x}$,

giving your answer as a single trigonometric term.

$1 - \cos 6x = 2 \sin^2 3x$

so the expression is $\frac{2 \sin^2 3x}{2 \sin 3x} = \sin 3x$

Answer: $\sin 3x$

8 Partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{-5x^2 - 26x - 17}{(x - 1)(x + 3)^2}$ in partial fractions.

$$\frac{-5x^2 - 26x - 17}{(x - 1)(x + 3)^2} \equiv \frac{A}{(x - 1)} + \frac{B}{(x + 3)} + \frac{C}{(x + 3)^2}$$

$-5x^2 - 26x - 17 \equiv A(x + 3)^2 + B(x - 1)(x + 3) + C(x - 1)$

$x = 1$ gives $A = -3$; $x = -3$ gives $C = -4$

Coefficients of x^2: $-5 = A + B$, so $B = -2$

Answer: $-\frac{3}{(x - 1)} - \frac{2}{(x + 3)} - \frac{4}{(x + 3)^2}$

9 Reduction to linear form (logarithms) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The variables x and y satisfy $y = ab^x$. A graph of ln y against x is a straight line with gradient ln 4 that passes through (0, ln 6).
Find the values of a and b.

$\ln y = \ln a + x \ln b$

gradient = $\ln b = \ln 4$, so $b = 4$

intercept $\ln a = \ln 6$, so $a = 6$

Answer: $a = 6$, $b = 4$

10 Integrating trigonometric functions using identities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact value of

$\int_0^{\frac{\pi}{12}} \tan^2 3x \, dx$.

Use $\tan^2 3x = \sec^2 3x - 1$

$\int \tan^2 3x \, dx = \frac{\tan 3x}{3} - x$

$= \left[\frac{\tan 3x}{3} - x\right]_0^{\frac{\pi}{12}}$

$= \frac{1}{3} - \frac{\pi}{12}$

Answer: $\frac{1}{3} - \frac{\pi}{12}$