Day 257 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $3x^2 - 6x$ in the form $a(x + p)^2 + q$.

(b) Write down the coordinates of the minimum point of $y = 3x^2 - 6x$.

Take out the factor 3: $3x^2 - 6x = 3(x^2 - 2x) - 0$

$= 3[(x - 1)^2 - 1] - 0$

$= 3(x - 1)^2 - 3$

$(x - 1)^2 \geq 0$ with equality at $x = 1$, so the minimum is -3 at $x = 1$.

Answer: (a) $3(x - 1)^2 - 3$ (b) $(1, -3)$

2 Total probability and Bayes (All boards)

R: it rains; L: Ali is late.

P(R) = 0.6, P(L | R) = 0.3, P(L | R') = 0.3.

(a) Find P(L).

(b) Find P(R | L).

(a) $P(L) = 0.6 \times 0.3 + 0.4 \times 0.3 = 0.18 + 0.12 = 0.3$

(b) $P(R \mid L) = \dfrac{0.6 \times 0.3}{0.3} = \dfrac{0.18}{0.3} = 0.600$

Answer: (a) 0.3 (b) $\frac{3}{5}$

3 Constant acceleration with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle has velocity $(-5\mathbf{i} - 2\mathbf{j})$ m s^{-1} at time $t = 0$ and moves with constant acceleration $(\mathbf{i} + 3\mathbf{j})$ m s^{-2}. Find its velocity and speed when $t = 2$.

$\mathbf{v} = \mathbf{u} + \mathbf{a}t = (-5\mathbf{i} - 2\mathbf{j}) + 2(\mathbf{i} + 3\mathbf{j})$

$= -3\mathbf{i} + 4\mathbf{j}$

Speed $= \sqrt{(-3^2 + 4^2)}$

Answer: $\mathbf{v} = (-3\mathbf{i} + 4\mathbf{j})$ m s^{-1}; speed 5 m s^{-1}

4 Fixed-point iteration (All boards)

(a) Show that $x^2 - x - 2 = 0$ can be written as $x = \sqrt{x + 2}$.

(b) Use $x_{n+1} = \sqrt{x_n + 2}$ with $x_0 = 3$ to find x_1, x_2, x_3 to 4 d.p.

(a) $x^2 = x + 2$, so $x = \sqrt{x + 2}$

(b) $x_1 = 2.2361$

$x_2 = 2.0582$

$x_3 = 2.0145$

Answer: $x_1 = 2.2361$, $x_2 = 2.0582$, $x_3 = 2.0145$

5 Equation of a transformed curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $y = x^3 - 2x^2 + x + 3$.

C is stretched by scale factor 2 parallel to the y-axis.

Find its new equation.

Stretch parallel to the y-axis: multiply y by the scale factor.

When stretched by scale factor 2 parallel to the y-axis: $y = 2x^3 - 4x^2 + 2x + 6$

Answer: $y = 2x^3 - 4x^2 + 2x + 6$

6 R cos(θ − α) form: express and maximum (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $4\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$. Give α to 1 d.p.

$R\cos\alpha = 4$, $R\sin\alpha = 3$

$R = \sqrt{4^2 + 3^2} = 5$

$\tan\alpha = \dfrac{3}{4}$, so $\alpha = 36.9°$

Answer: $5\cos(\theta - 36.9°)$

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7 Beam on two supports: reactions (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform beam AB of length 10 m and mass 10 kg rests horizontally on supports at C and D, where AC = 2 m and BD = 1 m.

A particle of mass 10 kg is placed on the beam 3 m from B. Take g = 9.8 m s^{-2}.

Find the reactions at C and D.

Moments about C: $R_D \times 7 = 10\,g \times 3 + 10\,g \times 5$

$R_D = 784 \div 7$

Resolve vertically: $R_C + R_D = 20\,g$

Answer: R_C = 84 N; R_D = 112 N

8 Finding mean and standard deviation from two probabilities (All boards)

The scores on a test are normally distributed.

P(X < 35) = 0.1587 and P(X > 50) = 0.0228.

Find the mean and standard deviation (3 s.f.).

$z_1 = -0.9998$ and $z_2 = 1.999$

$35 = \mu + (-0.9998)\sigma$ and $50 = \mu + 1.999\sigma$

Subtract: $15 = 2.999\,\sigma$, so $\sigma = 5.00$

$\mu = 35 - (-0.9998)(5.002) = 40.0$

Answer: μ = 40.0, σ = 5.00

9 Tangent to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle C has equation $(x + 2)^2 + (y - 3)^2 = 20$.

The point P(−6, 5) lies on C.

Find an equation of the tangent to C at P, in the form ax + by + c = 0 with integer a, b, c.

Centre of C = (−2, 3). Gradient of radius CP = $\frac{2}{-4} = -\frac{1}{2}$

The tangent is perpendicular to the radius: gradient = 2

$y - 5 = 2(x + 6)$

Multiply out and collect terms: $2x - y + 17 = 0$

Answer: 2x − y + 17 = 0

10 Arc length and sector area (radians) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 13 cm and arc length 23.4 cm.

Find (a) the angle of the sector in radians, (b) the area of the sector.

$s = r\theta$, so $\theta = \frac{23.4}{13} = 1.8$

Area = $\frac{1}{2}r^2\theta = \frac{1}{2} \times 13^2 \times 1.8 = 152.1$

Answer: (a) 1.8 rad (b) 152.1 cm^2