Day 256 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Solving exponential equations using logarithms (All boards)

Solve $2^x = 3^{x+1}$, giving your answer to 3 s.f.

$x \ln 2 = (x + 1) \ln 3$

$x(\ln 2 - \ln 3) = 1 \ln 3$

$x = \frac{1 \ln 3}{\ln 2 - \ln 3} = -2.710$

Answer: $x = -2.71$

2 Algebraic division (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the quotient and the remainder when

$-x^3 + 8x^2 - 8x - 5$ is divided by $(x - 2)$.

$-x^2 \times (x - 2) = -x^3 + 2x^2$, leaving $6x^2 - 8x - 5$

$6x \times (x - 2) = 6x^2 - 12x$, leaving $4x - 5$

$4 \times (x - 2) = 4x - 8$, leaving 3

Quotient $-x^2 + 6x + 4$, remainder 3

Answer: quotient $-x^2 + 6x + 4$, remainder 3

3 Integrating composite functions (reverse chain rule) (All boards)

Find (a) $\int 3 \cos 4x \, dx$,

(b) the exact value of $\int_0^{\frac{\pi}{8}} 3 \cos 4x \, dx$.

(a) Reverse the chain rule and divide by the coefficient of x: $\frac{3 \sin 4x}{4} + c$

(b) $\left[\frac{3 \sin 4x}{4}\right]_0^{\frac{\pi}{8}}$

$= \frac{3}{4}$

Answer: (a) $\frac{3 \sin 4x}{4} + c$ (b) $\frac{3}{4}$

4 Coded data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

10 values of x are coded using $y = \frac{(x - 20)}{5}$.

$\Sigma y = 18$ and $\Sigma y^2 = 212$.

Find the mean and standard deviation of x (3 s.f.).

Mean of $y = 18 \div 10 = 1.800$

Mean of $x = 20 + 5 \times 1.800 = 29.0$

Variance of $y = \frac{212}{10} - (1.800)^2 = 17.96$

SD of y = 4.238, so SD of $x = 5 \times 4.238 = 21.2$

Answer: mean 29.0, standard deviation 21.2

5 Integration by substitution (given substitution) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use the substitution $u = x^2 + 1$ to find the exact value of

$\int_0^1 \frac{2x}{x^2 + 1} \, dx$.

$\frac{du}{dx} = 2x$, so $2x \, dx = du$

Limits: $x = 0$ gives $u = 1$, $x = 1$ gives $u = 2$

$\int_1^2 \frac{1}{u} \, du = [\ln u]_1^2$

$= \ln 2$

Answer: $\ln 2$

Day 256 **A Level Maths: B Tier 10-a-day: Model Solutions**

6 Car towing a trailer (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car of mass 1500 kg tows a trailer of mass 250 kg along a straight horizontal road by a light horizontal tow bar. The resistances are 300 N on the car and 150 N on the trailer.
The engine's driving force is 3600 N. Find the acceleration and the tension in the tow bar.

Whole system: $3600 - 300 - 150 = 1750\,a$
$a = 1.8$
Trailer: $T - 150 = 250 \times 1.8$
Answer: $a = 1.8$ m s^{-2}; $T = 600$ N

7 Normal curve and shaded area (All boards)

$X \sim N(50, 10^2)$. The shaded area on the diagram represents the probability that X lies between two values.
Find this probability to 3 significant figures.

Shaded area = $P(55.00 < X < 65.00)$
$= P(X < 65.00) - P(X < 55.00)$
$= 0.9332 - 0.6915 = 0.242$
Answer: 0.242

8 Projectile at an angle: time and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is kicked from horizontal ground with speed 24.5 m s^{-1} at an angle θ above the horizontal, where $\tan\theta = \frac{4}{3}$.

Take $g = 9.8$ m s^{-2}.
Find (a) the time of flight, (b) the horizontal range.

$\cos\theta = 3/5$, $\sin\theta = 4/5$: horizontal 14.7, vertical 19.6
(a) $0 = 19.6\,t - 4.9t^2$, $t = 19.6 \div 4.9$
(b) Range $= 14.7 \times 4$
Answer: (a) 4 s (b) 58.8 m

9 Quadratics in a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the equation $5^{2x} - 3\times5^{x} - 10 = 0$.

Let $y = 5^x$. Then $5^{2x} = y^2$, so $y^2 - 3y - 10 = 0$
$(y - 5)(y + 2) = 0$
$y = 5$ or $y = -2$
$5^x > 0$ for all x, so $y = -2$ is rejected. $5^x = 5$ gives $x = 1$.
Answer: $x = 1$

10 Equation of a line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A and B have coordinates $(-1, 0)$ and $(2, -3)$.
(a) Find the midpoint of AB.
(b) Find the line AB as $ax + by + c = 0$, with integer a, b, c.

(a) Midpoint $= \left(\frac{-1+2}{2}, \frac{0+(-3)}{2}\right) = \left(\frac{1}{2}, -\frac{3}{2}\right)$

(b) Gradient $= \frac{-3}{3} = -1$

$y = -(x + 1)$
Multiply out and collect terms: $x + y + 1 = 0$
Answer: (a) $\left(\frac{1}{2}, -\frac{3}{2}\right)$ (b) $x + y + 1 = 0$