**Day 255** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 1 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^2 + 6x + 3$ for $x \geq -5$.
Find the range of f.

$f(x) = (x + 3)^2 - 6$, with minimum at $x = -3$
$x = -3$ is in the domain, so the minimum value is $-6$.
**Answer: $f(x) \geq -6$**

## 2 Constant acceleration (suvat) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A boat moves in a straight line with constant acceleration. Its initial speed is 1 m $s^{-1}$, its final speed is 2.5 m $s^{-1}$ and it travels 5.25 m.
Find the time taken.

$s = \frac{1}{2}(u + v)t$
$5.25 = \frac{1}{2} \times (1 + 2.5) \times t$
$t = 10.5 \div 3.5$
**Answer: 3 s**

## 3 Equation of a circle from given information (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line segment AB is a diameter of a circle, where A is $(-3, -6)$ and B is $(-7, -4)$.
Find an equation of the circle in the form
$(x - a)^2 + (y - b)^2 = k$.

Centre = midpoint of AB = $(-5, -5)$
$\text{Radius}^2 = (-3 - (-5))^2 + (-6 - (-5))^2 = 5$
**Answer: $(x + 5)^2 + (y + 5)^2 = 5$**

## 4 Product rule (All boards)

Given that $y = (x + 1)e^{3x}$,
find $\frac{dy}{dx}$, and its value when $x = 0$.

Product rule with $u = x + 1$, $v = e^{3x}$
$u' = 1$, $v' = 3e^{3x}$
$\frac{dy}{dx} = u'v + uv' = (3x + 4)e^{3x}$
When $x = 0$: $\frac{dy}{dx} = 4$
**Answer: $\frac{dy}{dx} = (3x + 4)e^{3x}$; value 4**

## 5 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|-x + 3| < -2x - 1$.

Critical values: $-x + 3 = -2x - 1$ gives $x = -4$; $-x + 3 = -(-2x - 1)$ gives $x = \frac{2}{3}$
Sketch $y = |-x + 3|$ and $y = -2x - 1$, and compare the graphs at $x = -4$ and $x = \frac{2}{3}$.
$|-x + 3| < -2x - 1$ when $x < -4$
**Answer: $x < -4$**

## 6 Reduction to linear form (logarithms) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The variables x and y satisfy $y = ax^n$. A graph of ln y against ln x is a straight line with gradient 3 that passes through (0, ln 4).
Find the values of a and n.

$\ln y = \ln a + n \ln x$
gradient = $n = 3$
intercept $\ln a = \ln 4$, so $a = 4$
**Answer: $a = 4$, $n = 3$**

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## 7 Normal approximation to the binomial (AQA, OCR, CIE)

X ~ B(60, 0.2). Use a normal approximation to find P(X ≥ 14), to 3 significant figures.

Y ~ N(np, np(1 − p)) = N(12.00, 9.600), so σ = $\sqrt{9.600}$ = 3.098
Use a continuity correction.
P(X ≥ 14) ≈ P(Y > 13.5) = P(Z > 0.4841) = 0.314
**Answer: 0.314**

## 8 Intersection of two lines in 3D (CIE)

Line $l_1$: $\mathbf{r} = \begin{pmatrix}3\\0\\0\end{pmatrix} + \lambda\begin{pmatrix}2\\-1\\3\end{pmatrix}$.

Line $l_2$: $\mathbf{r} = \begin{pmatrix}-5\\5\\-9\end{pmatrix} + \mu\begin{pmatrix}3\\-2\\3\end{pmatrix}$.

Show that $l_1$ and $l_2$ intersect and find the point.

i: 3 + 2λ = −5 + 3μ
j: 0 + (−1)λ = 5 + (−2)μ
Solving i and j: λ = −1, μ = 2
k check: 0 + 3(−1) = −3 and −9 + 3(2) = −3, equal

Point: $\begin{pmatrix}1\\1\\-3\end{pmatrix}$

**Answer:** $\begin{pmatrix}\mathbf{1}\\\mathbf{1}\\\mathbf{-3}\end{pmatrix}$

## 9 Projectile: speed and direction at time t (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is projected horizontally at 392 m $s^{-1}$ from a point high above the ground.
Take g = 9.8 m $s^{-2}$.
Find the speed and the direction of motion 30 s later, to 1 d.p. below the horizontal.

Vertical speed = 9.8 × 30 = 294
Speed = $\sqrt{(392^2 + 294^2)}$ = 490
tan α = 294 ÷ 392, α = 36.9°
**Answer: 490 m $s^{-1}$ at 36.9° below horizontal**

## 10 Finding mean and standard deviation from two probabilities (All boards)

The scores on a test are normally distributed.
P(X < 75) = 0.1587 and P(X > 87.5) = 0.0668.
Find the mean and standard deviation (3 s.f.).

$z_1$ = −0.9998 and $z_2$ = 1.500
75 = μ + (−0.9998)σ and 87.5 = μ + 1.500σ
Subtract: 12.5 = 2.500 σ, so σ = 5.00
μ = 75 − (−0.9998)(5.000) = 80.0
**Answer: μ = 80.0, σ = 5.00**