Day 254 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \sqrt{2x - 4} - 4$.
(a) Find the largest possible domain of f.
(b) Find the range of f.

The expression under the root must be non-negative: $2x - 4 \geq 0$
$2x \geq 4$ gives $x \geq 2$
$\sqrt{2x - 4} \geq 0$, so $f(x) \geq -4$
Answer: (a) $x \geq 2$ (b) $f(x) \geq -4$

2 Solving exponential equations using logarithms (All boards)

Solve $e^{2x} = 47$, giving your answer to 3 s.f.

Take natural logs: $2x = \ln 47$
$x = \frac{\ln 47}{2} = 1.925$
Answer: $x = 1.93$

3 Rough horizontal surface: finding μ (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sledge of mass 5 kg is pulled along rough horizontal ground by a horizontal force of 17.25 N. It accelerates at 1 m s^{-2}.
Take $g = 9.8$ m s^{-2}.
Find the coefficient of friction between the sledge and the ground.

$R = mg = 5 \times 9.8 = 49$
Resultant: $17.25 - F = 5 \times 1$, so $F = 12.25$
$F = \mu R$: $\mu = 12.25 \div 49$
Answer: 0.25

4 Two independent binomials (All boards)

$X \sim B(11, 0.5)$ and $Y \sim B(7, 0.1)$ are independent.
Find $P(X = 5 \text{ and } Y \leq 1)$, to 3 significant figures.

$P(X = 5) = 0.2256$
$P(Y \leq 1) = 0.8503$
Independent, so multiply: $0.2256 \times 0.8503 = 0.192$
Answer: 0.192

5 Differentiating position vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A model boat moves so that its position vector relative to O at time t s is $\mathbf{r} = (3t^2)\mathbf{i} + (t^3 - 3t)\mathbf{j}$ metres.
Find the velocity when t = 4, and the speed at that time.

$\mathbf{v} = d\mathbf{r}/dt = (6t)\mathbf{i} + (3t^2 - 3)\mathbf{j}$
$t = 4$: $\mathbf{v} = (24)\mathbf{i} + (45)\mathbf{j}$
Speed $= \sqrt{(24^2 + 45^2)}$
Answer: $\mathbf{v} = (24)\mathbf{i} + (45)\mathbf{j}$ m s^{-1}; speed 51 m s^{-1}

6 Simplify using double angle formulae (All boards)

Simplify $\frac{1 - \cos 2x}{\sin 2x}$,
giving your answer as a single trigonometric term.

$1 - \cos 2x = 2\sin^2 x$ and $\sin 2x = 2 \sin x \cos x$
so the expression is $\frac{2\sin^2 x}{2 \sin x \cos x} = \tan x$
Answer: $\tan x$

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7 Intersection of a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = -x^2 + 6x + 6$ and the line $y = 6x + 5$ meet at the points A and B.
Find the coordinates of A and B, and the exact length AB.

$-x^2 + 6x + 6 = 6x + 5$
$-x^2 + 1 = 0$, so $-(x + 1)(x - 1) = 0$
$x = -1$ or $x = 1$
$y = 6x + 5$: $y = -1$ and $y = 11$
$AB^2 = 2^2 + 12^2 = 148$
Answer: A(−1, −1), B(1, 11), AB = $2\sqrt{37}$

8 Tangents and normals to exponential and log curves (All boards)

The curve C has equation $y = (3x - 3)e^{4x}$.
Find the equation of the normal to C at the point where $x = 0$, in the form $y = mx + c$.

$\frac{dy}{dx} = 3(4x - 3)e^{4x}$

At $x = 0$: $y = -3$ and $\frac{dy}{dx} = -9$

Normal gradient $= -1 \div (-9) = \frac{1}{9}$

$y + 3 = \frac{1}{9}x$, so $y = \frac{1}{9}x - 3$

Answer: $y = \frac{1}{9}x - 3$

9 Discriminant problems (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The equation $x^2 + 2kx + (k + 2) = 0$ has no real roots.
Find the possible values of k.

$b^2 - 4ac = (2k)^2 - 4(k + 2) = 4k^2 - 4k - 8$
$= 4(k^2 - k - 2) = 4(k - 2)(k + 1)$
Need $b^2 - 4ac < 0$, so $-1 < k < 2$.
Answer: $-1 < k < 2$

10 Discrete random variable (AQA, OCR, MEI, Eduqas, CIE)

$P(X = x) = k(6 - x)$ for $x = 1, 2, ..., 5$.
(a) Find k.
(b) Find E(X) and Var(X).

(a) $k(5 + 4 + 3 + 2 + 1) = 1$, so $15k = 1$ and $k = \frac{1}{15}$

(b) $E(X) = 1 \times \frac{1}{3} + 2 \times \frac{4}{15} + 3 \times \frac{1}{5} + 4 \times \frac{2}{15} + 5 \times \frac{1}{15} = \frac{7}{3}$

$E(X^2) = 7$, so $Var(X) = 7 - (\frac{7}{3})^2 = \frac{14}{9}$

Answer: (a) $k = \frac{1}{15}$ (b) $E(X) = \frac{7}{3}$, $Var(X) = \frac{14}{9}$