Day 253 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Binomial expansion (positive integer power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In the binomial expansion of $(1 + kx)^n$, where n is a positive integer, the coefficient of x is 8 and the coefficient of x^2 is 28.
Find the values of n and k.

$nk = 8$, so $k = \frac{8}{n}$

$\frac{n(n-1)}{2}k^2 = 28$, so $\frac{n(n-1)}{2} \times \frac{64}{n^2} = 28$

$\frac{(n-1) \times 64}{2n} = 28$, so $64(n-1) = 56n$

$n = 8$, and $k = 1$

Answer: n = 8, k = 1

2 Braking distance and time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train travelling at 14 m s^{-1} brakes and comes to rest with constant deceleration 2 m s^{-2}.
Find (a) the distance travelled while braking, (b) the time taken to stop.

(a) $0 = 14^2 - 2 \times 2 \times s$

$s = 196 \div 4 = 49$

(b) $0 = 14 - 2t$, so $t = 14 \div 2$

Answer: (a) 49 m (b) 7 s

3 Inverse trigonometric functions (AQA, Edexcel, OCR, MEI, Eduqas)

Find the exact value of $\arctan(\tan \frac{5\pi}{6})$.

$\tan \frac{5\pi}{6} = -\frac{\sqrt{3}}{3}$

$\arctan(-\frac{\sqrt{3}}{3})$ must lie in $(-\frac{\pi}{2}, \frac{\pi}{2})$, so the value is $-\frac{\pi}{6}$

Answer: $-\frac{\pi}{6}$

4 Differentiating displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves on a straight line. Its displacement from O at time t seconds is $s = 4t^3 + 4t^2 + 7t$ metres.
Find its velocity and acceleration when t = 4.

$v = ds/dt = 12t^2 + 8t + 7$

$a = dv/dt = 24t + 8$

$t = 4$: $v = 231$, $a = 104$

Answer: v = 231 m s^{-1}, a = 104 m s^{-2}

5 Discriminant problems (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The equation $3x^2 + kx + 27 = 0$ has equal roots.
Find the possible values of k.

Equal roots: $b^2 - 4ac = 0$

$k^2 - 4 \times 3 \times 27 = 0$

$k^2 = 324$

$k = \pm 18$

Answer: k = ±18

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6 Rates of change (connected rates) (All boards)

The radius of a circular oil slick increases at 2 cm per second.
Find the rate at which its area is increasing when r = 8 cm, in terms of π.

$A = \pi r^2$, so $\frac{dA}{dr} = 2\pi r$

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt} = 2\pi r \times 2$

When r = 8: $\frac{dA}{dt} = 32\pi$

Answer: 32π cm^2/s

7 Binomial expansion (any power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(1 - x)^{-1}$ in ascending powers of x up to and including the term in x^3.
(b) State the range of values of x for which the expansion is valid.

$(1 + ax)^n = 1 + n(ax) + \frac{n(n-1)}{2!}(ax)^2 + \frac{n(n-1)(n-2)}{3!}(ax)^3 + ...$

Here n = −1 and a = −1:
coefficients: x: 1, x^2: 1, x^3: 1
Valid when |x| < 1, that is |x| < 1
Answer: (a) $1 + x + x^2 + x^3 + ...$ (b) $|x| < 1$

8 Integrals of the form f′(x)/f(x) (All boards)

Find the exact value of

$\int_0^{\frac{\pi}{2}} \frac{2\cos x}{\sin x + 3}\, dx$,

giving your answer in the form a ln b.

The numerator is a multiple of the derivative of the denominator

$\int \frac{2\cos x}{\sin x + 3}\, dx = 2\ln(\sin x + 3)$

$= [2\ln(\sin x + 3)]_0^{\frac{\pi}{2}}$

$= \ln(\frac{16}{9})$

Answer: $\ln(\frac{16}{9})$

9 Normal distribution probabilities (All boards)

The length X cm of a metal rod is normal with mean 250, s.d. 0.5.
Find, to 3 s.f., the probability a value is
(a) more than 249.2,
(b) between 249.2 and 249.5.

$X \sim N(250, 0.5^2)$
(a) $P(X > 249.2) = 0.945$
(b) $P(249.2 < X < 249.5) = 0.1587 - 0.05480 = 0.104$
Answer: (a) 0.945 (b) 0.104

10 Coded data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

9 values of x are coded using $y = \frac{(x - 200)}{5}$.

$\Sigma y = 4$ and $\Sigma y^2 = 102$.
Find the mean and standard deviation of x (3 s.f.).

Mean of y = 4 ÷ 9 = 0.4444
Mean of x = 200 + 5 × 0.4444 = 202

Variance of $y = \frac{102}{9} - (0.4444)^2 = 11.14$

SD of y = 3.337, so SD of x = 5 × 3.337 = 16.7
Answer: mean 202, standard deviation 16.7