Day 252 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Constant velocity: position vector (AQA, Edexcel, OCR, MEI, Eduqas)

A drone moves with constant velocity $(-3\mathbf{i} + 2\mathbf{j})$ m s^{-1}. At time t = 0 its position vector relative to a fixed origin O is $(-7\mathbf{i} - 2\mathbf{j})$ m.
Find (a) its position vector when t = 6, (b) the value of t when it crosses the line through O in the direction of $\mathbf{i}$.

(a) $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t = (-7\mathbf{i} - 2\mathbf{j}) + 6(-3\mathbf{i} + 2\mathbf{j}) = (-25\mathbf{i} + 10\mathbf{j})$
(b) $\mathbf{j}$ component zero: $-2 + 2t = 0$, so $t = 1$
Answer: (a) $(-25\mathbf{i} + 10\mathbf{j})$ m (b) t = 1 s

2 Proof by deduction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $(4n + 1)^2 - (4n - 1)^2$ is a multiple of 16 for all integer values of n.

$(4n + 1)^2 = 16n^2 + 8n + 1$
$(4n - 1)^2 = 16n^2 - 8n + 1$
Difference $= 16n = 16 \times n$
n is an integer, so this is a multiple of 16 for every integer n.
Answer: difference = 16n, a multiple of 16

3 Solving exponential equations using logarithms (All boards)

Solve $2^{2x-1} = 169$, giving your answer to 3 s.f.

Take logs: $(2x - 1)\ln 2 = \ln 169$
$2x - 1 = \dfrac{\ln 169}{\ln 2} = 7.4009$
$x = 4.200$
Answer: x = 4.20

4 Probability without replacement (All boards)

A bag has 7 red and 7 blue counters; two are taken without replacement.
(a) Find P(same colour).
(b) Given they are the same colour, find P(both red).

(a) $\text{P(red red)} = \dfrac{7}{14} \times \dfrac{6}{13} = \dfrac{3}{13}$
$\text{P(blue blue)} = \dfrac{7}{14} \times \dfrac{6}{13} = \dfrac{3}{13}$
$\text{P(same)} = \dfrac{3}{13} + \dfrac{3}{13} = \dfrac{6}{13}$
(b) $\text{P} = \dfrac{3}{13} \div \dfrac{6}{13} = \dfrac{1}{2}$
Answer: (a) $\dfrac{6}{13}$ (b) $\dfrac{1}{2}$

5 Rates of change (connected rates) (All boards)

The radius of a circular oil slick increases at 0.2 cm per second.
Find the rate at which its area is increasing when r = 4 cm, in terms of π.

$A = \pi r^2$, so $\dfrac{dA}{dr} = 2\pi r$
$\dfrac{dA}{dt} = \dfrac{dA}{dr} \times \dfrac{dr}{dt} = 2\pi r \times 0.2$
When r = 4: $\dfrac{dA}{dt} = \dfrac{8\pi}{5}$
Answer: $\dfrac{8\pi}{5}$ cm^2/s

6 Finding the equation of a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sketch of the curve y = f(x), where f is a cubic. The curve meets the axes at the labelled points. Find f(x) in factorised form.

x-intercepts at x = −4, −3 and −1, so y = k(x + 4)(x + 3)(x + 1)
The curve passes through (0, 12): k × 4 × 3 × 1 = 12
So k = 1
Answer: f(x) = (x + 4)(x + 3)(x + 1)

7 Histogram bar heights (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A class of width 20, frequency 40, is a bar 5 cm wide, 4 cm tall.
(a) Find the bar height for width 20, frequency 30.
(b) Find the frequency of a 2 cm by 2.5 cm bar.

Area of bar ∝ frequency: 5 × 4 = 20 cm^2 represents 40
so 1 unit of frequency = 0.5 cm^2
(a) Width = 5 cm, area = 0.5 × 30 = 15 cm^2, height = 15 ÷ 5 = 3 cm
(b) Area = 2 × 2.5 = 5 cm^2, frequency = 5 ÷ 0.5 = 10
Answer: (a) 3 cm (b) 10

8 Binomial expansion (any power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(1 - x)^{\frac{3}{2}}$ in ascending powers of x up to and including the term in x^3.
(b) State the range of values of x for which the expansion is valid.

$$(1 + ax)^n = 1 + n(ax) + \frac{n(n-1)}{2!}(ax)^2 + \frac{n(n-1)(n-2)}{3!}(ax)^3 + \ldots$$

Here $n = \frac{3}{2}$ and a = −1:

coefficients: x: $-\frac{3}{2}$, x^2: $\frac{3}{8}$, x^3: $\frac{1}{16}$

Valid when |x| < 1, that is |x| < 1

Answer: (a) $1 - \frac{3}{2}x + \frac{3}{8}x^2 + \frac{1}{16}x^3 + \ldots$ (b) $|x| < 1$

9 Speed and direction from a velocity vector (AQA, Edexcel, OCR, MEI, Eduqas)

A drone moves with constant velocity $(-12\mathbf{i} + 16\mathbf{j})$ m s^{-1}, where **i** and **j** are unit vectors east and north. Find its speed and the acute angle its direction makes with **i**, to 1 d.p.

Speed = $\sqrt{(-12^2 + 16^2)} = 20$
$\tan\theta = 16 \div 12$
Answer: 20 m s^{-1}; 53.1°

10 Double angle values from a given ratio (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sin\theta = \frac{12}{13}$ and $0 < \theta < 90°$,

find the exact value of $\tan 2\theta$.

$\cos\theta = \pm\sqrt{1 - (\frac{12}{13})^2} = \frac{5}{13}$ (θ is acute)

$\sin 2\theta = \frac{120}{169}$ and $\cos 2\theta = -\frac{119}{169}$

$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = -\frac{120}{119}$

Answer: $-\frac{120}{119}$