Day 251 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 3D vectors: magnitude, distance and unit vectors (All boards)

Points A and B have position vectors $\begin{pmatrix}-2\\3\\4\end{pmatrix}$ and $\begin{pmatrix}4\\5\\7\end{pmatrix}$.

Find AB and the distance AB.

$\text{AB} = \text{OB} - \text{OA} = \begin{pmatrix}6\\2\\3\end{pmatrix}$

$|\text{AB}| = \sqrt{6^2 + 2^2 + 3^2} = \sqrt{49} = 7$

Answer: AB $= \begin{pmatrix}\mathbf{6}\\\mathbf{2}\\\mathbf{3}\end{pmatrix}$**, distance 7**

2 Tangent to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle C has equation $(x + 1)^2 + (y + 2)^2 = 50$.
The point P$(-8, -3)$ lies on C.
Find an equation of the tangent to C at P, in the form $ax + by + c = 0$ with integer a, b, c.

Centre of C = $(-1, -2)$. Gradient of radius CP = $\frac{-1}{-7} = \frac{1}{7}$

The tangent is perpendicular to the radius: gradient = -7
$y + 3 = -7(x + 8)$
Multiply out and collect terms: $7x + y + 59 = 0$
Answer: 7x + y + 59 = 0

3 Integration by substitution (given substitution) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use the substitution $u = x^2 + 3$ to find the exact value of

$\int_0^6 \frac{x}{\sqrt{x^2 + 3}}\,dx$.

$\frac{du}{dx} = 2x$, so $x\,dx = \frac{1}{2}\,du$
Limits: $x = 0$ gives $u = 3$, $x = 6$ gives $u = 39$
$\int_3^{39} \frac{1}{2}u^{-\frac{1}{2}}\,du = [\sqrt{u}]_3^{39}$
$= \sqrt{39} - \sqrt{3}$
Answer: $\sqrt{\mathbf{39}} - \sqrt{\mathbf{3}}$

4 Recurrence relations (AQA, Edexcel, OCR, MEI, Eduqas)

A sequence is defined by $u_{n+1} = -2u_n + 4$, $u_1 = 6$.
(a) Find u_2 and u_3.
(b) Find $\sum_{r=1}^{4} u_r$.

(a) $u_2 = -2 \times 6 + 4 = -8$
$u_3 = -2 \times (-8) + 4 = 20$
(b) $u_4 = -2 \times 20 + 4 = -36$
Sum $= 6 + (-8) + 20 + (-36) = -18$
Answer: (a) $\mathbf{u_2 = -8}$**,** $\mathbf{u_3 = 20}$ **(b)** $\mathbf{-18}$

5 Car towing a trailer (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car of mass 800 kg tows a trailer of mass 200 kg along a straight horizontal road by a light horizontal tow bar. The resistances are 300 N on the car and 200 N on the trailer.
The engine's driving force is 2000 N. Find the acceleration and the tension in the tow bar.

Whole system: $2000 - 300 - 200 = 1000\,a$
$a = 1.5$
Trailer: $T - 200 = 200 \times 1.5$
Answer: $a = 1.5$ m s^{-2}; $T = 500$ N

6 Binomial expansion (any power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(1 + 2x)^{-\frac{1}{3}}$ in ascending powers of x up to and including the term in x^3.
(b) State the range of values of x for which the expansion is valid.

$(1 + ax)^n = 1 + n(ax) + \frac{n(n-1)}{2!}(ax)^2 + \frac{n(n-1)(n-2)}{3!}(ax)^3 + \ldots$

Here $n = -\frac{1}{3}$ and $a = 2$:

coefficients: x: $-\frac{2}{3}$, x^2: $\frac{8}{9}$, x^3: $-\frac{112}{81}$

Valid when $|2x| < 1$, that is $|x| < \frac{1}{2}$

Answer: (a) $1 - \frac{2}{3}x + \frac{8}{9}x^2 - \frac{112}{81}x^3 + \ldots$ (b) $|x| < \frac{1}{2}$

7 Inverse trigonometric functions (AQA, Edexcel, OCR, MEI, Eduqas)

Find the exact value of $\arcsin(\sin \frac{3\pi}{4})$.

$\sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$

$\arcsin(\frac{\sqrt{2}}{2})$ must lie in $[-\frac{\pi}{2}, \frac{\pi}{2}]$, so the value is $\frac{\pi}{4}$

Answer: $\frac{\pi}{4}$

8 Rough inclined plane: sliding down (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 3 kg is released from rest on a rough plane inclined at angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$.

The coefficient of friction is 0.25. Take g = 9.8 m s^{-2}. Find the acceleration down the plane, and the distance moved in 4 s.

$\cos \alpha = 4/5$, $\sin \alpha = 3/5$
$R = mg \cos \alpha$, $F = \mu R$
$ma = mg \sin \alpha - \mu mg \cos \alpha$, so $a = 9.8(3/5 - 0.25 \times 4/5)$
$s = \frac{1}{2}at^2 = \frac{1}{2} \times 3.92 \times 4^2$
Answer: 3.92 m s^{-2}; 31.36 m

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9 Continuous random variable: median (CIE)

The continuous random variable X has pdf $f(x) = \frac{2x}{100}$ for $0 \leq x \leq 10$, and 0 otherwise.

(a) Find $P(X > 4)$.

(b) Find the median of X, to 3 significant figures.

$F(x) = \int_0^x f(t)\,dt = \frac{x^2}{100}$

(a) $P(X > 4) = 1 - F(4) = \frac{21}{25}$

(b) $F(m) = 0.5$: $\frac{m^2}{100} = \frac{1}{2}$

$m = 7.07$

Answer: (a) $\frac{21}{25}$ (b) 7.07

10 Normal approximation to the binomial (AQA, OCR, CIE)

$X \sim B(80, 0.2)$. Use a normal approximation to find $P(17 \leq X \leq 20)$, to 3 significant figures.

$Y \sim N(np, np(1-p)) = N(16.00, 12.80)$, so $\sigma = \sqrt{12.80} = 3.578$

Use a continuity correction.

$P(17 \leq X \leq 20) \approx P(16.5 < Y < 20.5) = 0.340$

Answer: 0.340