Day 250 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Inequalities with exponents and logs (All boards)

Solve $2^x > 213$, giving your answer to 3 s.f.

Take logs: $x \ln 2 > \ln 213$

$x > \frac{\ln 213}{\ln 2} = 7.735$

Answer: $x > 7.73$

2 Speed and direction from a velocity vector (AQA, Edexcel, OCR, MEI, Eduqas)

A drone moves with constant velocity $(18\mathbf{i} + 24\mathbf{j})$ m s^{-1}, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors east and north.
Find its speed and the acute angle its direction makes with $\mathbf{i}$, to 1 d.p.

Speed $= \sqrt{(18^2 + 24^2)} = 30$

$\tan\theta = 24 \div 18$

Answer: 30 m s^{-1}; 53.1°

3 Basic trig equations (degrees) (All boards)

Solve $\tan x = -\sqrt{3}$ for $0 \le x \le 360°$.

Principal value $x = -60°$

tan has period 180°, so add 180°

Answer: $x = 120°, 300°$

4 Parametric to Cartesian (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = t^2$, $y = 4t$, $t \in \mathbb{R}$.
Find a Cartesian equation of the curve.

From $y = 4t$: $t = \frac{y}{4}$

$x = (\frac{y}{4})^2 = \frac{y^2}{16}$

So $y^2 = 16x$

Answer: $y^2 = 16x$

5 Pulling at an angle: smooth surface (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sledge of mass 4 kg is pulled across smooth horizontal ice by a rope with tension 26 N, inclined at α above the horizontal, where $\tan\alpha = \frac{5}{12}$.

Take $g = 9.8$ m s^{-2}.
Find (a) the acceleration of the sledge, (b) the normal reaction of the ice.

$\cos\alpha = 12/13$, $\sin\alpha = 5/13$

(a) Horizontal: $26 \times 12/13 = 4a$

(b) Vertical: $R + 26 \times 5/13 = 4 \times 9.8$

Answer: (a) 6 m s^{-2} (b) 29.2 N

6 Triangle formed by two lines and an axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Line l_1 has equation $x - 2y = 0$ and line l_2 has equation $3x - 8y = -6$.
The lines meet at P. Find the area of the triangle formed by l_1, l_2 and the x-axis.

Solve simultaneously: P = (6, 3)

l_1 meets $y = 0$ at $x = 0$, and l_2 meets $y = 0$ at $x = -2$

Base = 2, height = 3

Area $= \frac{1}{2} \times$ base $\times$ height $= 3$

Answer: 3

7 Volume of revolution about the x-axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The region R is bounded by the curve $y = e^{3x}$, the x-axis and the lines $x = 0$ and $x = 1$.
R is rotated through 360° about the x-axis. Find the exact volume generated.

$V = \pi \int_0^1 y^2\,dx = \pi \int_0^1 e^{6x}\,dx$

$= \pi[\frac{e^{6x}}{6}]_0^1$

$= -\frac{\pi(1 - e^6)}{6}$

Answer: $-\frac{\pi(1 - e^6)}{6}$

8 Coded data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

7 values of x are coded using $y = \frac{(x - 20)}{5}$.

$\Sigma y = 6$ and $\Sigma y^2 = 210$.
Find the mean and standard deviation of x (3 s.f.).

Mean of $y = 6 \div 7 = 0.8571$
Mean of $x = 20 + 5 \times 0.8571 = 24.3$

Variance of $y = \frac{210}{7} - (0.8571)^2 = 29.27$

SD of y = 5.410, so SD of $x = 5 \times 5.410 = 27.0$
Answer: mean 24.3, standard deviation 27.0

9 Partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{-x + 6}{(2x - 3)(x - 3)}$ in partial fractions.

$\frac{-x + 6}{(2x - 3)(x - 3)} \equiv \frac{A}{(2x - 3)} + \frac{B}{(x - 3)}$, so $-x + 6 \equiv A(x - 3) + B(2x - 3)$

$x = \frac{3}{2}$: $\frac{9}{2} = A \times (-\frac{3}{2})$, so $A = -3$

$x = 3$: $3 = B \times 3$, so $B = 1$

Answer: $-\frac{3}{(2x - 3)} + \frac{1}{(x - 3)}$

10 Finding mean and standard deviation from two probabilities (All boards)

The masses (kg) of parcels are normally distributed.
$P(X < 96) = 0.1587$ and $P(X > 104) = 0.1587$.
Find the mean and standard deviation (3 s.f.).

$z_1 = -0.9998$ and $z_2 = 0.9998$

$96 = \mu + (-0.9998)\sigma$ and $104 = \mu + 0.9998\sigma$

Subtract: $8 = 2.000\,\sigma$, so $\sigma = 4.00$

$\mu = 96 - (-0.9998)(4.001) = 100$

Answer: $\mu = 100$, $\sigma = 4.00$