Day 25 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Constant acceleration (suvat) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist moves in a straight line with constant acceleration. Its initial speed is 0 m s^{-1}, its final speed is 50 m s^{-1} and it travels 250 m.
Find the acceleration.

$v^2 = u^2 + 2as$
$50^2 = 0^2 + 2 \times a \times 250$
$2500 = 500\ a$
Answer: 5 m s^{-2}

2 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-x^2 - 2x \leq -3$.

Rearrange: $-x^2 - 2x + 3 \leq 0$
Factorise: $-(x - 1)(x + 3) \leq 0$
Critical values: $x = -3$ and $x = 1$
Sketch the $\cap$-shaped parabola: $x \leq -3$ or $x \geq 1$
Answer: $x \leq -3$ or $x \geq 1$

3 Tangents and normals to exponential and log curves (All boards)

The curve C has equation $y = (x + 2)e^{4x}$.
Find the equation of the normal to C at the point where $x = 0$, in the form $y = mx + c$.

$\frac{dy}{dx} = (4x + 9)e^{4x}$

At $x = 0$: $y = 2$ and $\frac{dy}{dx} = 9$

Normal gradient $= -1 \div 9 = -\frac{1}{9}$

$y - 2 = -\frac{1}{9}x$, so $y = -\frac{1}{9}x + 2$

Answer: $y = -\frac{1}{9}x + 2$

4 Simplifying surds (AQA, Edexcel, OCR, MEI, Eduqas)

Simplify $\frac{3}{\sqrt{3}} - \sqrt{27}$, giving your answer in the form $k\sqrt{3}$.

$\frac{3}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = 1\sqrt{3}$
$\sqrt{27} = 3\sqrt{3}$
$1\sqrt{3} - 3\sqrt{3} = -2\sqrt{3}$
Answer: $-2\sqrt{3}$

5 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Packet masses are normal, s.d. 5 g. Claimed mean 50 g; 50 packets have mean 49 g.
Test at the 5% level whether the true mean differs from 50.

$H_0: \mu = 50$, $H_1: \mu \neq 50$. Under H_0, $\overline{X} \sim N(50, \frac{5^2}{50})$

$z = \frac{49 - 50}{5/\sqrt{50}} = -1.414$
$P(Z < -1.414) = 0.0786 > 0.025$
Do not reject H_0: there is insufficient evidence that the mean has changed
Answer: do not reject H_0: insufficient evidence of a change in the mean

6 Equation of a circle from given information (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line segment AB is a diameter of a circle, where A is (6, 1) and B is (4, −3).
Find an equation of the circle in the form
$(x - a)^2 + (y - b)^2 = k$.

Centre = midpoint of AB = (5, −1)
Radius^2 = $(6 - 5)^2 + (1 - (-1))^2 = 5$
Answer: $(x - 5)^2 + (y + 1)^2 = 5$

7 Speed-time graph: find the maximum speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift travels between floors in 40 s. The speed-time graph is a trapezium, with the maximum speed V m s^{-1} held from t = 5 to t = 25.
The floors are 180 m apart. Find V, and the deceleration in the final stage.

Area = ½ × (40 + 20) × V = 180
60 V = 360 so V = 6
Deceleration = 6 ÷ 15
Answer: V = 6; deceleration 0.4 m s^{-2}

8 Exponential growth and decay models (All boards)

A car's value, £V, after t years is given by $V = 20000 \times 0.9^t$.
Find (a) the value after 3 years, (b) when it first falls below £10000, to 3 s.f.

(a) $V = 20000 \times 0.9^3 = 14580$
(b) $0.9^t < 0.5$, so $t \ln 0.9 < \ln 0.5$
$\ln 0.9 < 0$, so the inequality reverses: $t > \frac{\ln 0.5}{\ln 0.9} = 6.579$
Answer: (a) £14580 (b) t > 6.58 years

9 Continuous random variable: median (CIE)

The continuous random variable X has pdf $f(x) = \frac{2x}{36}$ for $0 \le x \le 6$, and 0 otherwise.
(a) Find P(X > 5).
(b) Find the median of X, to 3 significant figures.

$F(x) = \int_0^x f(t)\,dt = \frac{x^2}{36}$
(a) $P(X > 5) = 1 - F(5) = \frac{11}{36}$
(b) $F(m) = 0.5$: $\frac{m^2}{36} = \frac{1}{2}$
m = 4.24
Answer: (a) $\frac{11}{36}$ (b) 4.24

10 Inequalities with exponents and logs (All boards)

£1000 is invested at 20% compound interest per year.
Find the least whole number of years for the investment to exceed £5000.

$1000 \times 1.2^n > 5000$, so $1.2^n > 5$
$n > \frac{\ln 5}{\ln 1.2} = 8.827$
The least whole number is n = 9
Answer: 9 years