Day 249 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Newton's second law: lift and tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 600 kg is raised and lowered by a vertical cable. The lift accelerates upwards at 1.5 m s^{-2}.
Take g = 9.8 m s^{-2}.
Find the tension in the cable.

Resolve vertically (up positive): $T - 600 \times 9.8 = 600 \times 1.5$
$T = 600(9.8 + 1.5)$
Answer: 6780 N

2 Binomial probabilities (All boards)

X ~ B(27, 0.1). Find, to 3 significant figures,
(a) $P(X \geq 4)$
(b) $P(1 \leq X \leq 4)$

(a) $P(X \geq 4) = 1 - P(X \leq 3)$
(a) = 0.282
(b) $P(1 \leq X \leq 4) = P(X \leq 4) - P(X \leq 0)$
(b) = 0.815
Answer: (a) 0.282 (b) 0.815

3 Rates of change (connected rates) (All boards)

The radius of a circular oil slick increases at 0.25 cm per second.
Find the rate at which its area is increasing when r = 12 cm, in terms of π.

$A = \pi r^2$, so $\frac{dA}{dr} = 2\pi r$
$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt} = 2\pi r \times 0.25$
When r = 12: $\frac{dA}{dt} = 6\pi$
Answer: 6π cm^2/s

4 Beam on two supports: reactions (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform beam AB of length 4 m and mass 10 kg rests horizontally on supports at C and D, where AC = 1 m and BD = 0.5 m.
A particle of mass 10 kg is placed on the beam 1 m from B. Take g = 9.8 m s^{-2}.
Find the reactions at C and D.

Moments about C: $R_D \times 2.5 = 10\,g \times 1 + 10\,g \times 2$
$R_D = 294 \div 2.5$
Resolve vertically: $R_C + R_D = 20\,g$
Answer: R_C = 78.4 N; R_D = 117.6 N

5 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^2 - 2x + 7$ for $x \leq 0$.
Find the range of f.

$f(x) = (x - 1)^2 + 6$, with minimum at x = 1
x = 1 is not in the domain; f is decreasing on the domain, so the least value is f(0) = 7.
Answer: $f(x) \geq 7$

6 Solving exponential equations using logarithms (All boards)

Solve $2^x = 5^{x+1}$, giving your answer to 3 s.f.

$x \ln 2 = (x + 1) \ln 5$
$x(\ln 2 - \ln 5) = 1 \ln 5$
$x = \frac{1 \ln 5}{\ln 2 - \ln 5} = -1.756$
Answer: $x = -1.76$

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7 Using a binomial expansion to estimate (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(1 - 3x)^{\frac{1}{3}}$ up to the x^2 term.
(b) Use x = 0.01 to estimate $\sqrt[3]{0.97}$ to 5 decimal places.

(a) $(1 + ax)^n = 1 + n(ax) + \frac{n(n-1)}{2}(ax)^2 + ...$ with $n = \frac{1}{3}$, $a = -3$:

$= 1 - x - x^2$
(b) $1 - 3 \times 0.01 = 0.97$, so substitute x = 0.01:
$1 - 1 \times 0.01 - 1 \times 0.01^2 = 0.98990$
Answer: (a) $1 - x - x^2 + ...$ (b) 0.98990

8 Normal approximation to the binomial (AQA, OCR, CIE)

X ~ B(50, 0.6). Use a normal approximation to find $P(X \leq 25)$, to 3 significant figures.

$Y \sim N(np, np(1-p)) = N(30.00, 12.00)$, so $\sigma = \sqrt{12.00} = 3.464$
Use a continuity correction.
$P(X \leq 25) \approx P(Y < 25.5) = P(Z < -1.299) = 0.0970$
Answer: 0.0970

9 Equations with sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\cot^2 x - 4 \operatorname{cosec} x + 4 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p.

Use $\cot^2 x = \operatorname{cosec}^2 x - 1$: $\operatorname{cosec}^2 x - 4 \operatorname{cosec} x + 3 = 0$
$(\operatorname{cosec} x - 1)(\operatorname{cosec} x - 3) = 0$, so $\operatorname{cosec} x = 1$ or 3
$\operatorname{cosec} x = 1$, so $\sin x = 1$: $x = 90.0°$
$\operatorname{cosec} x = 3$, so $\sin x = \frac{1}{3}$: $x = 19.5°, 160.5°$
Answer: x = 19.5°, 90.0°, 160.5°

10 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 4x - 3$ and $g(x) = x^2 + 6$ for $x \in \mathbb{R}$.
(a) Find fg(x).
(b) Solve gf(x) = 87.

$fg(x) = f(x^2 + 6) = 4(x^2 + 6) - 3 = 4x^2 + 21$
$gf(x) = (4x - 3)^2 + 6 = 87$
$(4x - 3)^2 = 81$, so $4x - 3 = \pm 9$
$x = -\frac{3}{2}$ or $x = 3$

Answer: (a) $fg(x) = 4x^2 + 21$ (b) $x = -\frac{3}{2}$ or $x = 3$