Day 248 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Resultant of perpendicular forces (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Two forces of magnitude 10 N and 24 N act at right angles on a particle.
Find the magnitude of the resultant, and the angle it makes with the 10 N force, to 1 d.p.

Magnitude = $\sqrt{(10^2 + 24^2)} = \sqrt{676}$
$\tan \alpha = 24 \div 10$
Answer: 26 N at 67.4°

2 Triangle formed by two lines and an axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Line l_1 has equation $7x - 4y = -21$ and line l_2 has equation $7x + 5y = 42$.
The lines meet at P. Find the area of the triangle formed by l_1, l_2 and the x-axis.

Solve simultaneously: P = (1, 7)
l_1 meets $y = 0$ at $x = -3$, and l_2 meets $y = 0$ at $x = 6$
Base = 9, height = 7
Area = $\frac{1}{2} \times$ base $\times$ height = $\frac{63}{2}$
Answer: $\mathbf{\frac{63}{2}}$

3 Product rule (All boards)

Given that $y = (x + 2)e^x$,
find $\frac{dy}{dx}$, and its value when $x = 0$.

Product rule with $u = x + 2$, $v = e^x$
$u' = 1$, $v' = e^x$
$\frac{dy}{dx} = u'v + uv' = (x + 3)e^x$
When $x = 0$: $\frac{dy}{dx} = 3$
Answer: $\mathbf{\frac{dy}{dx} = (x + 3)e^x}$**; value 3**

4 Lift: weighing scales reading (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A woman of mass 90 kg stands on weighing scales in a lift. The lift decelerates at 0.25 m s^{-2} while moving downwards.
Take g = 9.8 m s^{-2}.
Find the force exerted on the woman by the scales.

The acceleration is upwards, magnitude 0.25
Taking up as positive: $R - 90 \times 9.8 = 90 \times 0.25$
$R = 90(9.8 + 0.25)$
Answer: 904.5 N

5 Vectors in a triangle (proof and ratio) (All boards)

In triangle OAB, OA = a and OB = b. M lies on AB with AM : MB = 1 : 2, and N lies on OB with ON : NB = 1 : 1.
Find OM and NM in terms of a and b.

AB = b − a, so AM = $\frac{1}{3}$(b − a)
OM = a + AM = $\frac{2a}{3} + \frac{b}{3}$
ON = $\frac{b}{2}$, so NM = OM − ON = $\frac{2a}{3} - \frac{b}{6}$
Answer: OM = $\mathbf{\frac{2a}{3} + \frac{b}{3}}$**, NM** = $\mathbf{\frac{2a}{3} - \frac{b}{6}}$

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6 Proof using parity (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $n^3 + 7n + 2$ is even for all integers n.

$n^3 + 7n = n(n^2 + 7)$

If n is even, $n(n^2 + 7)$ is even.

If n is odd, n^2 is odd and $n^2 + 7$ is even, so $n(n^2 + 7)$ is even.

So $n^3 + 7n$ is always even, and adding 2 gives an even number.

Answer: expression is even

7 Binomial expected frequency (All boards)

Each of 5 people independently supports a team with probability 0.4; X is the number of supporters. This is done 100 times. Estimate the number of times that X = 3.

$X \sim B(5, 0.4)$

$P(X = 3) = {}^5C_3 \times (0.4)^3 \times (0.6)^2 = 0.2304$

Expected = $100 \times 0.2304 = 23.04$

Answer: about 23 times

8 Using a binomial expansion to estimate (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(1 + 2x)^{-2}$ up to the x^2 term.

(b) Use x = 0.01 to estimate $\frac{1}{1.02^2}$ to 5 decimal places.

(a) $(1 + ax)^n = 1 + n(ax) + \frac{n(n-1)}{2}(ax)^2 + \ldots$ with $n = -2$, $a = 2$:

$= 1 - 4x + 12x^2$

(b) $1 + 2 \times 0.01 = 1.02$, so substitute x = 0.01:

$1 - 4 \times 0.01 + 12 \times 0.01^2 = 0.96120$

Answer: (a) $1 - 4x + 12x^2 + \ldots$ (b) 0.96120

9 R cos(θ − α) form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $15\cos\theta + 8\sin\theta \equiv 17\cos(\theta - 28.1°)$, solve $15\cos\theta + 8\sin\theta = -0.5$ for $0 \leq \theta \leq 360°$. (1 d.p.)

$17\cos(\theta - 28.1°) = -0.5$, so $\cos(\theta - 28.1°) = \frac{-0.5}{17}$

$\theta - 28.1° = \pm 91.7°$

$\theta = 28.1° + 91.7° = 119.8°$

$\theta = 28.1° - 91.7° = -63.6°$, so $\theta = 296.4°$

Answer: θ = 119.8°, 296.4°

10 Normal curve and shaded area (All boards)

$X \sim N(50, 10^2)$. The shaded area on the diagram represents the probability that X lies between two values. Find this probability to 3 significant figures.

Shaded area = P(30.00 < X < 60.00)

= P(X < 60.00) − P(X < 30.00)

= 0.8413 − 0.02275 = 0.819

Answer: 0.819