Day 247 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Regression line (AQA, Edexcel, OCR, MEI, Eduqas)

For 12 pairs of values, $\bar{x} = 10$, $\bar{y} = 40$, $S_{xx} = 80$ and $S_{xy} = 200$.
(a) Find the regression line of y on x in the form $y = a + bx$.
(b) Estimate y when $x = 14$.

(a) $b = \frac{S_{xy}}{S_{xx}} = \frac{200}{80} = 2.5$

$a = \bar{y} - b\bar{x} = 40 - 2.5 \times 10 = 15$
$y = 15 + 2.5x$
(b) $y = 15 + 2.5 \times 14 = 50$
Answer: $y = 15 + 2.5x$; $y = 50$

2 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $3x^2 + 30x + 79$ in the form $a(x + p)^2 + q$.
(b) Write down the coordinates of the minimum point of $y = 3x^2 + 30x + 79$.

Take out the factor 3: $3x^2 + 30x + 79 = 3(x^2 + 10x) + 79$
$= 3[(x + 5)^2 - 25] + 79$
$= 3(x + 5)^2 + 4$
$(x + 5)^2 \geq 0$ with equality at $x = -5$, so the minimum is 4 at $x = -5$.
Answer: (a) $3(x + 5)^2 + 4$ (b) $(-5, 4)$

3 Sector and arc in terms of π (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 7 cm and area $\frac{49\pi}{12}$ cm^2.
Find the angle of the sector in degrees.

$\frac{1}{2}r^2\theta = \frac{49\pi}{12}$, so $\theta = \frac{2 \times \frac{49\pi}{12}}{7^2} = \frac{\pi}{6}$ radians

$\theta = \frac{\pi}{6} \times \frac{180}{\pi} = 30°$
Answer: 30°

4 Reading a histogram (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The histogram shows the times taken by a group of people to complete a task.
Estimate the number of people who took between 10 and 18 minutes.

Frequency = frequency density × class width.
10 to 15: $5 \times 5 = 25$
15 to 18: $3 \times 3 = 9$
Total = 34
Answer: 34

5 Constant acceleration, two stages of working (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moving in a straight line with constant acceleration increases its speed from 15 m s^{-1} to 23 m s^{-1} while covering 76 m.
Find (a) the acceleration, (b) the time taken.

(a) $v^2 = u^2 + 2as$: $529 = 225 + 152\,a$, so $a = 2$
(b) $v = u + at$: $23 = 15 + 2\,t$
$t = 8 \div 2 = 4$
Answer: (a) 2 m s^{-2} (b) 4 s

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6 Tangent to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle C has equation $(x - 4)^2 + (y + 1)^2 = 65$.
The point P(11, 3) lies on C.
Find an equation of the tangent to C at P, in the form ax + by + c = 0 with integer a, b, c.

Centre of C = (4, −1). Gradient of radius CP = $\frac{4}{7}$

The tangent is perpendicular to the radius: gradient = $-\frac{7}{4}$

$y - 3 = -\frac{7}{4}(x - 11)$

Multiply out and collect terms: $7x + 4y - 89 = 0$

Answer: $7x + 4y - 89 = 0$

7 Derivatives of tan, sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Differentiate y = 2 cot x with respect to x,

and find the exact gradient when $x = \frac{\pi}{4}$.

Use the standard results for tan, sec, cosec and cot with the chain rule

$\frac{dy}{dx} = -2\,\text{cosec}^2 x$

At $x = \frac{\pi}{4}$: $\frac{dy}{dx} = -4$

Answer: $\frac{dy}{dx} = -2\,\text{cosec}^2 x$; gradient -4

8 Partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{-3x + 3}{(2x - 3)(x - 2)}$ in partial fractions.

$\frac{-3x + 3}{(2x - 3)(x - 2)} \equiv \frac{A}{(2x - 3)} + \frac{B}{(x - 2)}$, so $-3x + 3 \equiv A(x - 2) + B(2x - 3)$

$x = \frac{3}{2}$: $-\frac{3}{2} = A \times (-\frac{1}{2})$, so A = 3

x = 2: −3 = B × 1, so B = −3

Answer: $\frac{3}{(2x - 3)} - \frac{3}{(x - 2)}$

9 Displacement from a v-t formula; average velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves along a straight line so that its displacement from O after t s is $s = 3t^3 + 3t^2$ metres.
Find (a) the average velocity over the first 4 seconds, (b) the velocity at t = 4.

(a) s(4) = 240, so average velocity = 240 ÷ 4

(b) $v = ds/dt = 9t^2 + 6t$, v(4) = 168

Answer: (a) 60 m s^{-1} (b) 168 m s^{-1}

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10 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix} 6 \\ 0 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 2 \\ 2 \end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $6 + 2\lambda = 4 + (-2)\mu$

j: $0 + (-1)\lambda = 0 + 2\mu$

Solving i and j: $\lambda = -2$, $\mu = 1$

k check: $-2 + (-2)(-2) = 2$ and $0 + 2(1) = 2$, equal

Point: $\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{2} \\ \mathbf{2} \\ \mathbf{2} \end{pmatrix}$