Day 246 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Simplifying surds (AQA, Edexcel, OCR, MEI, Eduqas)

Simplify $\frac{36}{\sqrt{6}} - \sqrt{54}$, giving your answer in the form $k\sqrt{6}$.

$\frac{36}{\sqrt{6}} = \frac{36\sqrt{6}}{6} = 6\sqrt{6}$

$\sqrt{54} = 3\sqrt{6}$

$6\sqrt{6} - 3\sqrt{6} = 3\sqrt{6}$

Answer: $3\sqrt{6}$

2 Parametric differentiation: tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations

$x = t^2$, $y = t^3$.

Find the gradient at $t = -1$ and the equation of the normal there.

$\frac{dx}{dt} = 2t$, $\frac{dy}{dt} = 3t^2$

Gradient $= \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = -\frac{3}{2}$ at $t = -1$

Point: $(1, -1)$; normal gradient $= \frac{2}{3}$

$y + 1 = \frac{2}{3}(x - 1)$, so $y = \frac{2}{3}x - \frac{5}{3}$

Answer: gradient $-\frac{3}{2}$; normal $y = \frac{2}{3}x - \frac{5}{3}$

3 Equation of a line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A and B have coordinates (3, 2) and (5, −1).

(a) Find the midpoint of AB.

(b) Find the line AB as $ax + by + c = 0$, with integer a, b, c.

(a) Midpoint $= (\frac{3+5}{2}, \frac{2+(-1)}{2}) = (4, \frac{1}{2})$

(b) Gradient $= \frac{-3}{2} = -\frac{3}{2}$

$y - 2 = -\frac{3}{2}(x - 3)$

Multiply out and collect terms: $3x + 2y - 13 = 0$

Answer: (a) $(4, \frac{1}{2})$ (b) $3x + 2y - 13 = 0$

4 Position vectors and dividing a line in a ratio (All boards)

A has position vector $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$ and B has position vector $\begin{pmatrix} 4 \\ -9 \end{pmatrix}$.

The point P lies on AB with AP : PB = 3 : 1.

Find the position vector of P.

$\text{AB} = \begin{pmatrix} 4 \\ -8 \end{pmatrix}$

$\text{AP} = \frac{3}{4}\text{AB} = \begin{pmatrix} 3 \\ -6 \end{pmatrix}$

$\text{OP} = \text{OA} + \text{AP} = \begin{pmatrix} 3 \\ -7 \end{pmatrix}$

Answer: $\begin{pmatrix} 3 \\ -7 \end{pmatrix}$

5 Venn diagram with independent events (All boards)

The Venn diagram shows probabilities for events A and B, which are independent.
Find the values of x and y.

P(A) = 0.3 + 0.1 = 0.4 and P(A ∩ B) = 0.1
Independent: P(A ∩ B) = P(A)P(B), so P(B) = 0.1 ÷ 0.4 = 0.25
x = P(B) − P(A ∩ B) = 0.25 − 0.1 = 0.15
y = 1 − (0.3 + 0.1 + 0.15) = 0.45
Answer: x = 0.15, y = 0.45

6 Perpendicular bisector (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is the point (8, 6) and B is the point (−3, −1).
Find the perpendicular bisector of AB as ax + by + c = 0, with integer a, b, c.

Midpoint of AB = $(\frac{5}{2}, \frac{5}{2})$

Gradient of AB = $\frac{-7}{-11} = \frac{7}{11}$

Perpendicular gradient = $-\frac{11}{7}$

$y - \frac{5}{2} = -\frac{11}{7}(x - \frac{5}{2})$, which gives 11x + 7y − 45 = 0
Answer: 11x + 7y − 45 = 0

7 Pulley system: speed on reaching the ground (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (3 kg) and B (7 kg) are joined by a light string over a smooth pulley. Both are held with the string taut, B is 0.25 m above the ground, then released.
Take g = 9.8 m s^{-2}.
Find the speed with which B hits the ground.

a = (7 − 3)g ÷ (7 + 3) = 3.92
v^2 = 0 + 2 × 3.92 × 0.25 = 1.96
v = 1.4
Answer: 1.4 m s^{-1}

8 Finding the mean or standard deviation (All boards)

A journey takes X minutes, normal with mean 500.
Given P(X < 505.9) = 0.975, find the standard deviation (3 s.f.).

z = 1.960, so $\frac{505.9 - 500}{\sigma} = 1.960$
σ = 5.9 ÷ 1.960 = 3.01
Answer: σ = 3.01

9 Integrating velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t seconds its velocity is v = $3t^2 - 5t$ m s^{-1}.
Find the distance moved between t = 2 and t = 4. (The particle does not change direction.)

$s = \int_2^4 (3t^2 - 5t)\,dt$

$= [\, t^3 - \frac{5}{2}t^2 \,]$ from t = 2 to t = 4

= 24 − −2
Answer: 26 m

10 Stationary points of exponential and log curves (All boards)

Find the coordinates of the stationary points of the curve $y = x^2e^{-x}$, and determine their nature.

$\frac{dy}{dx} = -x(x - 2)e^{-x} = 0$

$x = 0$ or $x = 2$

$\frac{d^2y}{dx^2} = (x^2 + 2 - 4x)e^{-x}$

At $x = 0$: $\frac{d^2y}{dx^2} = 2 > 0$, so a minimum

At $x = 2$: $\frac{d^2y}{dx^2} = -\frac{2}{e^2} < 0$, so a maximum

Answer: (0, 0) minimum; $(2, \frac{4}{e^2})$ maximum