Day 245 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Frequency table: mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table shows the number of siblings, x, of 46 students.
Calculate the mean and standard deviation (3 s.f.).

$\Sigma f = 46$, $\Sigma fx = 101$, $\Sigma fx^2 = 353$

Mean = $\frac{101}{46} = 2.20$

Variance = $\frac{353}{46} - (2.1957)^2 = 2.853$

Standard deviation = 1.69

Answer: mean 2.20, standard deviation 1.69

2 Parametric to Cartesian (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = 2t$, $y = \frac{4}{t}$, $t \neq 0$.

Find a Cartesian equation of the curve.

Multiply the two equations: $xy = 2t \times \frac{4}{t} = 8$

So $y = \frac{8}{x}$

Answer: $y = \frac{8}{x}$

3 Sector and arc in terms of π (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 6 cm and angle 270°.
Find, in terms of π, the arc length and the area of the sector.

$\theta = 270° = \frac{3\pi}{2}$ radians

Arc length = $r\theta = 9\pi$ cm

Area = $\frac{1}{2}r^2\theta = 27\pi$ cm^2

Answer: arc 9π cm, area 27π cm^2

4 Two particles over a pulley (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles of mass 6 kg and 9 kg hang on a light string over a smooth fixed pulley, as shown, and are released from rest.
Take $g = 9.8$ m s^{-2}.
Find the acceleration and the tension in the string.

Heavier: $9g - T = 9a$
Lighter: $T - 6g = 6a$
Add: $3g = 15a$, so $a = 1.96$
$T = 6(9.8 + 1.96)$

Answer: $a = 1.96$ m s^{-2}; $T = 70.56$ N

5 Factor theorem with an unknown (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^3 - 3x^2 + kx + 12$.
Given that $(x - 3)$ is a factor of $f(x)$,
(a) find the value of k,
(b) factorise $f(x)$ completely.

$f(3) = 0$: $27 - 27 + 3k + 12 = 0$
$k = -4$

$f(x) = (x - 3)(x^2 - 4)$
$= (x - 3)(x - 2)(x + 2)$

Answer: (a) $k = -4$ (b) $f(x) = (x - 3)(x - 2)(x + 2)$

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6 Exponential growth and decay models (All boards)

The mass of a radioactive sample, M grams, after t days is $M = 80\,e^{-0.15t}$.
Find (a) the half-life, (b) the rate of decrease in mass when t = 5, each to 3 s.f.

(a) $80\,e^{-0.15t} = 40$: $e^{-0.15t} = \frac{1}{2}$, so $t = \frac{\ln 2}{0.15} = 4.621$ days

(b) $\frac{dM}{dt} = -0.15 \times 80\,e^{-0.15t} = -12\,e^{-0.15t}$

At t = 5: $\frac{dM}{dt} = -12\,e^{-0.75} = -5.668$

so mass is decreasing at 5.67 g per day

Answer: (a) 4.62 days (b) 5.67 g/day

7 Inverse normal (All boards)

A journey takes X minutes, normal with mean 100, s.d. 12.
Find k (3 s.f.): (a) P(X < k) = 0.2,
(b) P(X > k) = 0.01.

(a) z = −0.8416, so k = 100 + (−0.8416) × 12 = 89.9
(b) P(X < k) = 0.99, z = 2.326, so k = 100 + 2.326 × 12 = 128

Answer: (a) 89.9 (b) 128

8 Equations with sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\cot^2 x - 3 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p.

Use $\cot^2 x = \operatorname{cosec}^2 x - 1$: $\operatorname{cosec}^2 x - 4 = 0$
(cosec x + 2)(cosec x − 2) = 0, so cosec x = −2 or 2

cosec x = −2, so $\sin x = -\frac{1}{2}$: x = 210.0°, 330.0°

cosec x = 2, so $\sin x = \frac{1}{2}$: x = 30.0°, 150.0°

Answer: x = 30.0°, 150.0°, 210.0°, 330.0°

9 Chord of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line y = 3x − 17 meets the circle $x^2 + (y + 2)^2 = 25$ at the points A and B. Find the coordinates of A and B, and the exact length of the chord AB.

Substitute y = 3x − 17: $x^2 + (3x - 15)^2 = 25$
$x^2 - 9x + 20 = 0$
(x − 5)(x − 4) = 0, so x = 4 or x = 5
y = 3x − 17: A = (4, −5), B = (5, −2)
$AB^2 = 1^2 + 3^2 = 10$

Answer: A(4, −5), B(5, −2), AB = $\sqrt{10}$

10 Beam about to tilt (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform plank AB of length 6 m and mass 20 kg rests horizontally on supports at C and D, where AC = 2 m and DB = 1.5 m.
A girl of mass 20 kg walks from D towards B. Find how far she is from D when the plank is about to tilt.

About to tilt about D: reaction at C is zero
Moments about D: 20 g × (4.5 − 3) = 20 g × x
20 × 1.5 = 20 x

Answer: 1.5 m