Day 244 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 3D vectors: magnitude, distance and unit vectors (All boards)

Points A and B have position vectors $\begin{pmatrix}-3\\-3\\2\end{pmatrix}$ and $\begin{pmatrix}-15\\0\\-2\end{pmatrix}$.

Find AB and the distance AB.

$\text{AB} = \text{OB} - \text{OA} = \begin{pmatrix}-12\\3\\-4\end{pmatrix}$

$|\text{AB}| = \sqrt{12^2 + 3^2 + 4^2} = \sqrt{169} = 13$

Answer: AB $= \begin{pmatrix}\mathbf{-12}\\\mathbf{3}\\\mathbf{-4}\end{pmatrix}$**, distance 13**

2 Expectation and variance of aX + b (AQA, OCR, MEI, Eduqas, CIE)

The random variable X has E(X) = 5 and Var(X) = 16.

Find (a) E(−2X + 5),

(b) Var(−2X + 5), (c) $\text{E}(X^2)$.

(a) $\text{E}(aX + b) = a\text{E}(X) + b = (-2) \times 5 + 5 = -5$

(b) $\text{Var}(aX + b) = a^2\text{Var}(X) = 4 \times 16 = 64$

(c) $\text{Var}(X) = \text{E}(X^2) - [\text{E}(X)]^2$, so $\text{E}(X^2) = 16 + 25 = 41$

Answer: (a) −5 (b) 64 (c) 41

3 Right-angled triangle on a grid (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C are (−1, 0), (−3, −1) and (−4, 1).

(a) Show that angle ABC = 90°.

(b) Find the area of triangle ABC.

(a) Gradient AB $= \frac{1}{2}$

Gradient BC $= \frac{2}{-1} = -2$

$\frac{1}{2} \times (-2) = -1$, so AB and BC are perpendicular.

(b) AB $= \sqrt{5}$ and BC $= \sqrt{5}$

Area $= \frac{1}{2} \times \text{AB} \times \text{BC} = \frac{5}{2}$

Answer: (b) area $= \mathbf{\frac{5}{2}}$

4 Parametric differentiation: tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations x = 6 cos t, y = 4 sin t.

Find $\frac{\text{dy}}{\text{dx}}$ in terms of t, and its exact value when $\text{t} = \frac{\pi}{6}$.

$\frac{\text{dx}}{\text{dt}} = -6 \sin \text{t}$, $\frac{\text{dy}}{\text{dt}} = 4 \cos \text{t}$

$\frac{\text{dy}}{\text{dx}} = \frac{\frac{\text{dy}}{\text{dt}}}{\frac{\text{dx}}{\text{dt}}} = -\frac{2 \cot \text{t}}{3}$

At $\text{t} = \frac{\pi}{6}$: $\frac{\text{dy}}{\text{dx}} = -\frac{2\sqrt{3}}{3}$

Answer: $\mathbf{\frac{dy}{dx} = -\frac{2 \cot t}{3}}$**; value** $\mathbf{-\frac{2\sqrt{3}}{3}}$

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5 Newton's second law: lift and tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 600 kg is raised and lowered by a vertical cable. The lift accelerates downwards at 0.25 m s^{-2}.
Take g = 9.8 m s^{-2}.
Find the tension in the cable.

Resolve vertically (up positive): $T - 600 \times 9.8 = -600 \times 0.25$
$T = 600(9.8 - 0.25)$
Answer: 5730 N

6 Tangent to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle C has equation $(x - 4)^2 + (y - 1)^2 = 65$.
The point P(8, 8) lies on C.
Find an equation of the tangent to C at P, in the form ax + by + c = 0 with integer a, b, c.

Centre of C = (4, 1). Gradient of radius CP = $\frac{7}{4}$

The tangent is perpendicular to the radius: gradient = $-\frac{4}{7}$

$y - 8 = -\frac{4}{7}(x - 8)$

Multiply out and collect terms: $4x + 7y - 88 = 0$
Answer: $4x + 7y - 88 = 0$

7 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Bottle volumes are normal, s.d. 6 ml. Set mean 250 ml; 50 bottles have mean 251.6 ml.
Test at the 5% level whether the true mean is greater than 250.

H_0: $\mu = 250$, H_1: $\mu > 250$. Under H_0, $\overline{X} \sim N(250, \frac{6^2}{50})$

$z = \frac{251.6 - 250}{6/\sqrt{50}} = 1.886$
$P(Z > 1.886) = 0.0297 < 0.05$
Reject H_0: there is sufficient evidence that the mean has increased
Answer: reject H_0: sufficient evidence of an increase in the mean

8 Differential equations in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cup of tea cools in a room at 15°C: $\frac{dT}{dt} = -k(T - 15)$. Initially T = 95 and after 10 minutes T = 50.
Find k to 3 s.f. and the time to cool to 25°C, to 3 s.f.

$\frac{1}{T - 15}\, dT = -k\, dt$, so $\ln(T - 15) = -kt + c$, i.e. $T = 15 + 80e^{-kt}$

t = 10: $50 = 15 + 80e^{-10k}$, so $e^{-10k} = \frac{7}{16}$

$k = \frac{1}{10} \ln \frac{80}{35} = 0.08267$

$25 = 15 + 80e^{-kt}$: $t = \frac{1}{k} \ln \frac{80}{10} = 25.15$

Answer: k = 0.0827, t = 25.2 minutes

9 Inclined plane: force to hold or move the particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 2 kg is held on a rough plane ($\tan\alpha = \frac{4}{3}$, $\mu = 0.75$) by a force P N up the plane, as shown.

Take g = 9.8 m s^{-2}. The block is about to move down the plane. Find P.

$R = mg\cos\alpha = 19.6 \times 3/5 = 11.76$
Friction (limiting) $F = \mu R = 8.82$
Moving down, friction acts up: $P + F = mg\sin\alpha$
$mg\sin\alpha = 15.68$
Answer: 6.86 N

10 Perpendicular lines and shortest distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line l has equation $y = x + 7$. The point P is (0, 1).

(a) Find the line through P perpendicular to l, as $ax + by + c = 0$ with integer a, b, c.

(b) Find the foot F and the exact distance PF.

(a) Gradient of l = 1, so the perpendicular has gradient -1

$y - 1 = -x$, giving $x + y - 1 = 0$

(b) Solve $y = x + 7$ with $x + y - 1 = 0$: F = $(-3, 4)$

$PF^2 = 3^2 + (-3)^2 = 18$

Answer: (a) $x + y - 1 = 0$ (b) F$(-3, 4)$, PF = $3\sqrt{2}$