Day 243 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Integrating composite functions (reverse chain rule) (All boards)

Find (a) $\int 2\sec^2 3x\,dx$,

(b) the exact value of $\int_0^{\frac{\pi}{12}} 2\sec^2 3x\,dx$.

(a) Reverse the chain rule and divide by the coefficient of x: $\frac{2\tan 3x}{3} + c$

(b) $\left[\frac{2\tan 3x}{3}\right]_0^{\frac{\pi}{12}}$

$= \frac{2}{3}$

Answer: (a) $\frac{2\tan 3x}{3} + c$ (b) $\frac{2}{3}$

2 Centre and radius of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $x^2 + y^2 + 2x + 10y + 17 = 0$.
Find the coordinates of its centre and its radius.

Complete the square in x and y:

$(x + 1)^2 - 1 + (y + 5)^2 - 25 + 17 = 0$

$(x + 1)^2 + (y + 5)^2 = 9$

Centre $(-1, -5)$, radius 3

Answer: centre $(-1, -5)$, radius 3

3 Pulling at an angle: smooth surface (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sledge of mass 2 kg is pulled across smooth horizontal ice by a rope with tension 5 N, inclined at α above the horizontal, where $\tan\alpha = \frac{4}{3}$.

Take g = 9.8 m s^{-2}.
Find (a) the acceleration of the sledge, (b) the normal reaction of the ice.

$\cos\alpha = 3/5$, $\sin\alpha = 4/5$

(a) Horizontal: $5 \times 3/5 = 2\,a$

(b) Vertical: $R + 5 \times 4/5 = 2 \times 9.8$

Answer: (a) 1.5 m s^{-2} (b) 15.6 N

4 Poisson distribution (CIE)

X ~ Po(3), modelling the number of flaws per metre of cloth.
Find, to 3 significant figures,
(a) P(X = 3)
(b) $P(X \leq 3)$

(a) $P(X = 3) = e^{-3} \times \frac{3^3}{3!} = 0.224$

(b) $P(X \leq 3) = P(X = 0) + \ldots + P(X = 3) = 0.647$

Answer: (a) 0.224 (b) 0.647

5 Projectile: speed and direction at time t (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is projected horizontally at 117.6 m s^{-1} from a point high above the ground.
Take g = 9.8 m s^{-2}.
Find the speed and the direction of motion 16 s later, to 1 d.p. below the horizontal.

Vertical speed $= 9.8 \times 16 = 156.8$

Speed $= \sqrt{(117.6^2 + 156.8^2)} = 196$

$\tan\alpha = 156.8 \div 117.6$, $\alpha = 53.1°$

Answer: 196 m s^{-1} at 53.1° below horizontal

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6 Integrals of the form f′(x)/f(x) (All boards)

Find the exact value of

$\int_0^{\frac{\pi}{2}} \frac{4\cos x}{\sin x + 1}\,dx$,

giving your answer in the form a ln b.

The numerator is a multiple of the derivative of the denominator

$\int \frac{4\cos x}{\sin x + 1}\,dx = 4\ln(\sin x + 1)$

$= [4\ln(\sin x + 1)]_0^{\frac{\pi}{2}}$

$= \ln 16$

Answer: ln 16

7 Finding the equation of a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sketch of the curve y = f(x), where f is a cubic. The curve meets the axes at the labelled points. Find f(x) in factorised form.

x-intercepts at $x = -3$, -2 and 1, so $y = k(x + 3)(x + 2)(x - 1)$

The curve passes through (0, 12): $k \times 3 \times 2 \times (-1) = 12$

So $k = -2$

Answer: f(x) = −2(x + 3)(x + 2)(x − 1)

8 Laws of logarithms: solving equations (All boards)

Solve $\log_3(x + 2) - \log_3(x - 2) = 1$.

$\log_3\left(\frac{x+2}{x-2}\right) = 1$, so $\frac{x+2}{x-2} = 3^1 = 3$

$x + 2 = 3(x - 2) = 3x - 6$

$2x = 8$, so $x = 4$

Answer: x = 4

9 Binomial two-tailed test (All boards)

A factory says a proportion 0.3 of its parts are defective. In a sample of 20 parts, 3 are defective.
Test, at the 5% significance level, whether the proportion is different from 0.3.

H_0: $p = 0.3$, H_1: $p \neq 0.3$. Under H_0, $X \sim B(20, 0.3)$

$P(X \leq 3) = 0.1071$

Two-tailed: compare with 0.025. 0.1071 > 0.025, so do not reject H_0

There is insufficient evidence that the proportion is different from 0.3

Answer: do not reject H_0: insufficient evidence that the proportion is different from 0.3

10 Right-angled triangle on a grid (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C are (−1, 2), (1, −2) and (5, 0).
(a) Show that angle ABC = 90°.
(b) Find the area of triangle ABC.

(a) Gradient AB = $\frac{4}{-2} = -2$

Gradient BC = $\frac{2}{4} = \frac{1}{2}$

$(-2) \times \frac{1}{2} = -1$, so AB and BC are perpendicular.

(b) AB = $2\sqrt{5}$ and BC = $2\sqrt{5}$

Area = $\frac{1}{2} \times AB \times BC = 10$

Answer: (b) area = 10