Day 242 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Normal distribution probabilities (All boards)

A journey takes X minutes, normal with mean 170, s.d. 0.5.
Find, to 3 s.f., the probability a value is
(a) less than 171,
(b) between 171 and 171.5.

$X \sim N(170, 0.5^2)$
(a) $P(X < 171) = 0.977$
(b) $P(171 < X < 171.5) = 0.9987 - 0.9772 = 0.0214$
Answer: (a) 0.977 (b) 0.0214

2 Lift: weighing scales reading (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A woman of mass 50 kg stands on weighing scales in a lift. The lift decelerates at 2 m s^{-2} while moving downwards.
Take g = 9.8 m s^{-2}.
Find the force exerted on the woman by the scales.

The acceleration is upwards, magnitude 2
Taking up as positive: $R - 50 \times 9.8 = 50 \times 2$
$R = 50(9.8 + 2)$
Answer: 590 N

3 Proof using parity (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $n^2 + 7n + 2$ is even for all integers n.

$n^2 + 7n = n(n + 7)$
As 7 is odd, n and n + 7 have opposite parity,
so one of them is even and the product n(n + 7) is even.
Even + 2 = even, so $n^2 + 7n + 2$ is even.
Answer: n(n + 7) is even, so the expression is even

4 Discrete random variable (AQA, OCR, MEI, Eduqas, CIE)

P(X = x) = kx for x = 1, 2, ..., 6.
(a) Find k.
(b) Find E(X) and Var(X).

(a) $k(1 + 2 + 3 + 4 + 5 + 6) = 1$, so $21k = 1$ and $k = \frac{1}{21}$
(b) $E(X) = 1 \times \frac{1}{21} + 2 \times \frac{2}{21} + 3 \times \frac{1}{7} + 4 \times \frac{4}{21} + 5 \times \frac{5}{21} + 6 \times \frac{2}{7} = \frac{13}{3}$
$E(X^2) = 21$, so $Var(X) = 21 - (\frac{13}{3})^2 = \frac{20}{9}$
Answer: (a) k = $\frac{1}{21}$ (b) E(X) = $\frac{13}{3}$, Var(X) = $\frac{20}{9}$

5 Factor theorem with an unknown (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^3 + 2x^2 + kx$.
Given that $(x - 1)$ is a factor of f(x),
(a) find the value of k,
(b) factorise f(x) completely.

f(1) = 0: 1 + 2 + k = 0
k = −3
$f(x) = (x - 1)(x^2 + 3x)$
$= (x - 1)x(x + 3)$
Answer: (a) k = −3 (b) f(x) = (x − 1)x(x + 3)

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6 Pulling at an angle: smooth surface (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sledge of mass 3 kg is pulled across smooth horizontal ice by a rope with tension 25 N, inclined at α above the horizontal, where $\tan\alpha = \frac{4}{3}$.

Take $g = 9.8$ m s^{-2}.
Find (a) the acceleration of the sledge, (b) the normal reaction of the ice.

$\cos\alpha = 3/5$, $\sin\alpha = 4/5$
(a) Horizontal: $25 \times 3/5 = 3a$
(b) Vertical: $R + 25 \times 4/5 = 3 \times 9.8$
Answer: (a) 5 m s^{-2} (b) 9.4 N

7 Vectors in a triangle (proof and ratio) (All boards)

In triangle OAB, OA = a and OB = b. M lies on AB with AM : MB = 1 : 3, and N lies on OB with ON : NB = 3 : 1.
Find OM and NM in terms of a and b.

AB = b − a, so AM = $\frac{1}{4}$(b − a)

OM = a + AM = $\frac{3a}{4} + \frac{b}{4}$

ON = $\frac{3b}{4}$, so NM = OM − ON = $\frac{3a}{4} - \frac{b}{2}$

Answer: OM = $\frac{3a}{4} + \frac{b}{4}$, NM = $\frac{3a}{4} - \frac{b}{2}$

8 Remainder theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 2x^3 + px^2 + qx - 9$.
When f(x) is divided by (x − 3) the remainder is 3, and when divided by (x + 1) the remainder is −17. Find p and q.

$f(3) = 3$: $54 + 9p + 3q - 9 = 3$, so $9p + 3q = -42$
$f(-1) = -17$: $-2 + p - q - 9 = -17$, so $p - q = -6$
Solving simultaneously: $p = -5$, $q = 1$
Answer: p = −5, q = 1

9 Double angle equations (All boards)

Solve $2\cos 2x = 7\cos x$ for $0 \le x \le 360°$,
giving answers to 1 d.p. where necessary.

Use the double angle formula: $4\cos^2 x - 2 - 7\cos x = 0$
$(4\cos x + 1)(\cos x - 2) = 0$
$\cos x = -\frac{1}{4}$: $x = 104.5°, 255.5°$
$\cos x = 2$ has no solutions
Answer: x = 104.5°, 255.5°

10 Integration by parts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use integration by parts to find the exact value of

$\int_0^2 xe^{-x}\,dx$.

Let $u = x$ and $\frac{dv}{dx} = e^{-x}$, so $\frac{du}{dx} = 1$ and $v = -e^{-x}$

Integral = $[uv] - \int v\frac{du}{dx}\,dx$

$= [-xe^{-x}]_0^2 - \int_0^2 -e^{-x}\,dx$

$= 1 - \frac{3}{e^2}$

Answer: $1 - \frac{3}{e^2}$