Day 241 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Triangle formed by two lines and an axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Line l_1 has equation $x - 7y = -6$ and line l_2 has equation $x - 4y = -3$.
The lines meet at P. Find the area of the triangle formed by l_1, l_2 and the x-axis.

Solve simultaneously: P = (1, 1)
l_1 meets y = 0 at $x = -6$, and l_2 meets y = 0 at $x = -3$
Base = 3, height = 1
Area = $\frac{1}{2} \times$ base $\times$ height = $\frac{3}{2}$

Answer: $\frac{3}{2}$

2 Tangents and normals to exponential and log curves (All boards)

The curve C has equation $y = \ln(2x^2 - 1) - 2x$.
Find the equation of the tangent to C at the point where x = 1, in the form y = mx + c.

$\frac{dy}{dx} = 2(\frac{2x}{2x^2 - 1} - 1)$

At x = 1: $y = -2$ and $\frac{dy}{dx} = 2$
$y + 2 = 2(x - 1)$, so $y = 2x - 4$
Answer: $y = 2x - 4$

3 Will it move? Limiting friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of mass 15 kg rests on a rough horizontal floor, with coefficient of friction 0.25. A horizontal force of 29 N is applied to the box.
Take g = 9.8 m s^{-2}.
Determine whether the box moves. Find the friction force, or the acceleration if it moves.

R = 15 × 9.8 = 147, maximum friction $\mu R = 0.25 \times 147 = 36.75$
29 < 36.75 so the box stays at rest
Friction balances the applied force: F = 29
Answer: Does not move; friction = 29 N

4 Probability without replacement (All boards)

A bag has 6 green and 8 yellow beads; two are taken without replacement.
(a) Find P(exactly one green).
(b) Find P(at least one green).

(a) P(green then yellow) + P(yellow then green) $= 2 \times \frac{6}{14} \times \frac{8}{13} = \frac{48}{91}$

(b) $1 - $ P(both yellow) $= 1 - \frac{8}{14} \times \frac{7}{13} = 1 - \frac{4}{13} = \frac{9}{13}$

Answer: (a) $\frac{48}{91}$ (b) $\frac{9}{13}$

5 Lift: weighing scales reading (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A woman of mass 80 kg stands on weighing scales in a lift. The lift decelerates at 0.25 m s^{-2} while moving upwards.
Take g = 9.8 m s^{-2}.
Find the force exerted on the woman by the scales.

The acceleration is downwards, magnitude 0.25
Taking up as positive: $R - 80 \times 9.8 = -80 \times 0.25$
$R = 80(9.8 - 0.25)$
Answer: 764 N

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6 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Packet masses are normal, s.d. 5 g. Claimed mean 250 g; 16 packets have mean 248.2 g.
Test at the 5% level whether the true mean differs from 250.

H_0: $\mu = 250$, H_1: $\mu \neq 250$. Under H_0, $\overline{X} \sim N(250, \frac{5^2}{16})$

$z = \frac{248.2 - 250}{5/\sqrt{16}} = -1.440$

$P(Z < -1.440) = 0.0749 > 0.025$

Do not reject H_0: there is insufficient evidence that the mean has changed

Answer: do not reject H_0: insufficient evidence of a change in the mean

7 Remainder theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 2x^3 + px^2 + qx + 8$.

When f(x) is divided by (x + 3) the remainder is −49, and when divided by (x − 1) the remainder is 7. Find p and q.

$f(-3) = -49$: $-54 + 9p - 3q + 8 = -49$, so $9p - 3q = -3$

$f(1) = 7$: $2 + p + q + 8 = 7$, so $p + q = -3$

Solving simultaneously: $p = -1$, $q = -2$

Answer: $p = -1$, $q = -2$

8 Derivatives of tan, sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Differentiate $y = 5 \sec 3x$ with respect to x,

and find the exact gradient when $x = \frac{\pi}{9}$.

Use the standard results for tan, sec, cosec and cot with the chain rule

$\frac{dy}{dx} = 15 \tan 3x \sec 3x$

At $x = \frac{\pi}{9}$: $\frac{dy}{dx} = 30\sqrt{3}$

Answer: $\frac{dy}{dx} = 15 \tan 3x \sec 3x$; gradient $30\sqrt{3}$

9 Binomial expansion (any power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(1 + 2x)^{-\frac{1}{2}}$ in ascending powers of x up to and including the term in x^3.

(b) State the range of values of x for which the expansion is valid.

$(1 + ax)^n = 1 + n(ax) + \frac{n(n-1)}{2!}(ax)^2 + \frac{n(n-1)(n-2)}{3!}(ax)^3 + \ldots$

Here $n = -\frac{1}{2}$ and $a = 2$:

coefficients: x: -1, x^2: $\frac{3}{2}$, x^3: $-\frac{5}{2}$

Valid when $|2x| < 1$, that is $|x| < \frac{1}{2}$

Answer: (a) $1 - x + \frac{3}{2}x^2 - \frac{5}{2}x^3 + \ldots$ (b) $|x| < \frac{1}{2}$

10 Sector and arc in terms of π (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 9 cm and area $\frac{81\pi}{5}$ cm^2.

Find the angle of the sector in degrees.

$\frac{1}{2}r^2\theta = \frac{81\pi}{5}$, so $\theta = \frac{2 \times \frac{81\pi}{5}}{9^2} = \frac{2\pi}{5}$ radians

$\theta = \frac{2\pi}{5} \times \frac{180}{\pi} = 72°$

Answer: 72°