Day 240 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Integrating composite functions (reverse chain rule) (All boards)

Find (a) $\int 6e^{4x}\,dx$,

(b) the exact value of $\int_0^1 6e^{4x}\,dx$.

(a) Reverse the chain rule and divide by the coefficient of x: $\frac{3e^{4x}}{2} + c$

(b) $[\frac{3e^{4x}}{2}]_0^1$

$= \frac{3e^4}{2} - \frac{3}{2}$

Answer: (a) $\frac{3e^{4x}}{2} + c$ (b) $\frac{3e^4}{2} - \frac{3}{2}$

2 Independent and mutually exclusive events (All boards)

A and B are mutually exclusive events with P(A) = 0.4 and P(A ∪ B) = 0.8.
(a) Find P(B).
(b) Find P(A' ∩ B').

(a) Mutually exclusive: P(A ∪ B) = P(A) + P(B)
P(B) = 0.8 − 0.4 = 0.4
(b) P(A' ∩ B') = 1 − P(A ∪ B) = 0.2
Answer: (a) 0.4 (b) 0.2

3 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

f(x) = 3x − 4 for $-4 \le x < -2$.
Find the range of f.

f is increasing, so the extreme values occur at the ends of the domain.
f(−4) = −16 and f(−2) = −10
x = −4 is included and x = −2 is excluded, so f = −16 is included and f = −10 is excluded.
Answer: $-16 \le f(x) < -10$

4 Compound angle formulae (exact values) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that sin A = $\frac{40}{41}$ and sin B = $\frac{4}{5}$, where A and B are acute,

find the exact value of sin(A + B).

cos A = $\frac{9}{41}$ and cos B = $\frac{3}{5}$
sin(A + B) = sin A cos B + cos A sin B
$= \frac{40}{41} \times \frac{3}{5} + \frac{9}{41} \times \frac{4}{5} = \frac{156}{205}$
Answer: $\frac{156}{205}$

5 Power, driving force and resistance (CIE)

A car of mass 800 kg moves along a horizontal road. The engine works at a constant rate of 24 kW. The resistance to motion is constant at 400 N.

Find the acceleration when the speed is 20 m s^{-1}.

P = Fv: F = 24000 ÷ 20 = 1200 N
F − R = ma: 1200 − 400 = 800 a
Answer: 1 m s^{-2}

6 Solving exponential equations using logarithms (All boards)

Solve $3^x = 53$, giving your answer to 3 s.f.

Take logs: x ln 3 = ln 53
$x = \frac{\ln 53}{\ln 3} = 3.614$
Answer: x = 3.61

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7 Overtaking problem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A police car starts from rest and accelerates at 2.5 m s^{-2}. At that moment it is passed by a speeding van travelling at constant speed 15 m s^{-1}.
Find how long it takes to catch up, and the distance travelled.

Same distance: $\frac{1}{2} \times 2.5 \times t^2 = 15\,t$
$t(1.25\,t - 15) = 0$, $t \neq 0$ so $t = 15 \div 1.25$
Distance $= 15 \times 12$
Answer: 12 s, 180 m

8 Binomial expansion (any power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(1 - 3x)^{-\frac{1}{3}}$ in ascending powers of x up to and including the term in x^3.
(b) State the range of values of x for which the expansion is valid.

$(1 + ax)^n = 1 + n(ax) + \frac{n(n-1)}{2!}(ax)^2 + \frac{n(n-1)(n-2)}{3!}(ax)^3 + \ldots$

Here $n = -\frac{1}{3}$ and $a = -3$:

coefficients: x: 1, x^2: 2, x^3: $\frac{14}{3}$

Valid when $|3x| < 1$, that is $|x| < \frac{1}{3}$

Answer: (a) $1 + x + 2x^2 + \frac{14}{3}x^3 + \ldots$ (b) $|x| < \frac{1}{3}$

9 Expectation and variance of aX + b (AQA, OCR, MEI, Eduqas, CIE)

The random variable X has E(X) = 10 and Var(X) = 16.
Find (a) E(4X + 3),
(b) Var(4X + 3), (c) $E(X^2)$.

(a) $E(aX + b) = aE(X) + b = 4 \times 10 + 3 = 43$
(b) $Var(aX + b) = a^2Var(X) = 16 \times 16 = 256$
(c) $Var(X) = E(X^2) - [E(X)]^2$, so $E(X^2) = 16 + 100 = 116$
Answer: (a) 43 (b) 256 (c) 116

10 Tangent to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle C has equation $(x - 2)^2 + (y + 1)^2 = 169$.
The point P(14, −6) lies on C.
Find an equation of the tangent to C at P, in the form ax + by + c = 0 with integer a, b, c.

Centre of C = (2, −1). Gradient of radius CP = $\frac{-5}{12} = -\frac{5}{12}$

The tangent is perpendicular to the radius: gradient = $\frac{12}{5}$

$y + 6 = \frac{12}{5}(x - 14)$

Multiply out and collect terms: $12x - 5y - 198 = 0$
Answer: $12x - 5y - 198 = 0$