Day 24 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

Items numbered 1 to 143; systematic sample of size 13.
(a) Find the sampling interval.
(b) The first item is number 9. Find the 5-th item.

(a) Interval = $143 \div 13 = 11$
(b) 5-th item = $9 + 4 \times 11 = 53$
Answer: interval 11; item number 53

2 Compound angle formulae (exact values) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sin A = \frac{15}{17}$ and $\cos B = \frac{12}{13}$, where A is obtuse and B is acute,
find the exact value of $\sin(A + B)$.

$\cos A = -\frac{8}{17}$ and $\sin B = \frac{5}{13}$
$\sin(A + B) = \sin A \cos B + \cos A \sin B$
$= \frac{15}{17} \times \frac{12}{13} + (-\frac{8}{17}) \times \frac{5}{13} = \frac{140}{221}$
Answer: $\frac{140}{221}$

3 Poisson distribution (CIE)

X ~ Po(3), modelling the number of emails received per hour.
Find, to 3 significant figures,
(a) P(X = 3)
(b) P(X ≥ 3)

(a) $P(X = 3) = e^{-3} \times \frac{3^3}{3!} = 0.224$
(b) $P(X \geq 3) = 1 - P(X \leq 2) = 0.577$
Answer: (a) 0.224 (b) 0.577

4 Sector and arc in terms of π (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 5 cm and angle 90°.
Find, in terms of π, the arc length and the area of the sector.

$\theta = 90° = \frac{\pi}{2}$ radians

Arc length $= r\theta = \frac{5\pi}{2}$ cm

Area $= \frac{1}{2}r^2\theta = \frac{25\pi}{4}$ cm^2

Answer: arc $\frac{5\pi}{2}$ cm, area $\frac{25\pi}{4}$ cm^2

5 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $\sum_{r=3}^{15}(4r - 4)$.

There are $15 - 3 + 1 = 13$ terms
First term = 8, last term = 56
$S = \frac{13}{2}(8 + 56) = 416$
Answer: 416

6 Projectile: speed and direction at time t (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A missile is projected horizontally at 137.2 m s^{-1} from a point high above the ground.
Take g = 9.8 m s^{-2}.
Find the speed and the direction of motion 48 s later, to 1 d.p. below the horizontal.

Vertical speed = 9.8 × 48 = 470.4
Speed = $\sqrt{(137.2^2 + 470.4^2)}$ = 490
tan α = 470.4 ÷ 137.2, α = 73.7°
Answer: 490 m s^{-1} at 73.7° below horizontal

7 Scalar product and angle between vectors (CIE)

$\mathbf{a} = \begin{pmatrix} -2 \\ 2 \\ 5 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix}$.
Find the angle between a and b, to 1 d.p.

$\mathbf{a}\cdot\mathbf{b} = (-2)(-2) + (2)(1) + (5)(-4) = -14$
$|\mathbf{a}| = \sqrt{33}$, $|\mathbf{b}| = \sqrt{21}$
$\cos\theta = \frac{-14}{\sqrt{33}\sqrt{21}} = -0.5318$
θ = 122.1°
Answer: 122.1°

8 Using a binomial expansion to estimate (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(1 + 3x)^{\frac{1}{2}}$ up to the x^2 term.
(b) Use x = 0.02 to estimate $\sqrt{1.06}$ to 5 decimal places.

(a) $(1 + ax)^n = 1 + n(ax) + \frac{n(n-1)}{2}(ax)^2 + \ldots$ with $n = \frac{1}{2}$, a = 3:

$= 1 + \frac{3}{2}x - \frac{9}{8}x^2$

(b) 1 + 3 × 0.02 = 1.06, so substitute x = 0.02:

$1 + \frac{3}{2} \times 0.02 - \frac{9}{8} \times 0.02^2 = 1.02955$

Answer: (a) $1 + \frac{3}{2}x - \frac{9}{8}x^2 + \ldots$ (b) 1.02955

9 Instantaneous rest and displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on a straight line from O. At time t s its velocity is $6t^2 - 42t + 60$ m s^{-1}.
Find the first time P is at instantaneous rest, and its distance from O then.

v = 0: 6(t − 2)(t − 5) = 0, so t = 2 or t = 5
First time t = 2

$s = \int_0^2 v\,dt = [\,2t^3 - 21t^2 + 60t\,]$ from 0 to 2

Answer: t = 2 s; 52 m

10 Modulus equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve |4x − 3| = |x + 3|.

Either 4x − 3 = x + 3 or 4x − 3 = −(x + 3)
4x − 3 = x + 3 gives x = 2
4x − 3 = −(x + 3) gives x = 0
Answer: x = 0 and x = 2