Day 239 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Simplifying surds (AQA, Edexcel, OCR, MEI, Eduqas)

Expand and simplify $(-3 + 2\sqrt{5})(4 - \sqrt{5})$, giving your answer in the form $a + b\sqrt{5}$.

$(-3 + 2\sqrt{5})(4 - \sqrt{5}) = -12 + 3\sqrt{5} + 8\sqrt{5} - 2\sqrt{5}\times\sqrt{5}$

$\sqrt{5}\times\sqrt{5} = 5$, so the expression $= -22 + 11\sqrt{5}$

Answer: $-22 + 11\sqrt{5}$

2 Locating roots: change of sign (All boards)

The function $f(x) = 2\cos x - \frac{1}{2}x$.

Show that $f(x) = 0$ has a root between $x = 1$ and $x = 1.5$ (x in radians).

$f(1) = 0.581$

$f(1.5) = -0.609$

f is continuous and changes sign, so there is a root in the interval

Answer: f(1) > 0 and f(1.5) < 0: sign change

3 Braking distance and time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car travelling at 16 m s^{-1} brakes and comes to rest with constant deceleration 2.5 m s^{-2}.
Find (a) the distance travelled while braking, (b) the time taken to stop.

(a) $0 = 16^2 - 2 \times 2.5 \times s$

$s = 256 \div 5 = 51.2$

(b) $0 = 16 - 2.5\,t$, so $t = 16 \div 2.5$

Answer: (a) 51.2 m (b) 6.4 s

4 Inverse normal (All boards)

A journey takes X minutes, normal with mean 30, s.d. 10.
Find k (3 s.f.): (a) $P(X < k) = 0.25$,
(b) $P(X > k) = 0.1$.

(a) $z = -0.6745$, so $k = 30 + (-0.6745) \times 10 = 23.3$

(b) $P(X < k) = 0.9$, $z = 1.282$, so $k = 30 + 1.282 \times 10 = 42.8$

Answer: (a) 23.3 (b) 42.8

5 Rod suspended by two vertical strings (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 3 m and mass 6 kg hangs horizontally from two vertical strings attached 1 m from A and 1 m from B.

A particle of mass 4 kg is attached 1.5 m from A. Take g = 9.8 m s^{-2}. Find the tension in each string.

Moments about the string near A: $T_2(1) = 6\,g(0.5) + 4\,g(0.5)$

$T_2 = 49$

$T_1 + T_2 = 10\,g$ so $T_1 = 98 - 49$

Answer: Near A: 49 N; near B: 49 N

6 Tangent to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle C has equation $(x - 2)^2 + (y - 1)^2 = 50$.
The point P(3, −6) lies on C.
Find an equation of the tangent to C at P, in the form $ax + by + c = 0$ with integer a, b, c.

Centre of C = (2, 1). Gradient of radius CP $= \frac{-7}{1} = -7$

The tangent is perpendicular to the radius: gradient $= \frac{1}{7}$

$y + 6 = \frac{1}{7}(x - 3)$

Multiply out and collect terms: $x - 7y - 45 = 0$

Answer: $x - 7y - 45 = 0$

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7 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{2x + 8}{(x + 1)(x + 3)}$ in partial fractions.

(b) Hence evaluate $\int_0^1 \frac{2x + 8}{(x + 1)(x + 3)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{2x + 8}{(x + 1)(x + 3)} \equiv \frac{A}{x + 1} + \frac{B}{x + 3}$, so $2x + 8 \equiv A(x + 3) + B(x + 1)$
$x = -1$: $A = 3$; $x = -3$: $B = -1$

(b) $\int_0^1 \left(\frac{3}{x + 1} + \frac{-1}{x + 3}\right) dx = [3\ln(x + 1) - \ln(x + 3)]_0^1$

$= 3\ln\frac{2}{1} - \ln\frac{4}{3} = \ln(6)$

Answer: (a) A = 3, B = −1 (b) ln(6)

8 Chord of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = x + 17$ meets the circle $(x - 1)^2 + (y - 3)^2 = 125$ at the points A and B. Find the coordinates of A and B, and the exact length of the chord AB.

Substitute $y = x + 17$: $(x - 1)^2 + (x + 14)^2 = 125$
$x^2 + 13x + 36 = 0$
$(x + 4)(x + 9) = 0$, so $x = -9$ or $x = -4$
$y = x + 17$: A = (−9, 8), B = (−4, 13)
$AB^2 = 5^2 + 5^2 = 50$

Answer: A(−9, 8), B(−4, 13), AB = $5\sqrt{2}$

9 Reduction to linear form (logarithms) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The variables x and y satisfy $y = Ae^{kx}$. A graph of ln y against x is a straight line through (0, 1.5) and (4, 2.7).
Find A to 3 s.f. and k.

$\ln y = \ln A + kx$, so $\ln A = 1.5$ and $A = e^{1.5} = 4.482$

k = gradient $= \frac{2.7 - 1.5}{4 - 0} = 0.3$

Answer: A = 4.48, k = 0.3

10 Finding mean and standard deviation from two probabilities (All boards)

The random variable X is normally distributed.
P(X < 30) = 0.1587 and P(X > 50) = 0.1587.
Find the mean and standard deviation (3 s.f.).

$z_1 = -0.9998$ and $z_2 = 0.9998$
$30 = \mu + (-0.9998)\sigma$ and $50 = \mu + 0.9998\sigma$
Subtract: $20 = 2.000\,\sigma$, so $\sigma = 10.0$
$\mu = 30 - (-0.9998)(10.00) = 40.0$

Answer: μ = 40.0, σ = 10.0