Day 238 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Particle in equilibrium: string and horizontal force (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 6 kg hangs in equilibrium on a light string inclined at α to the vertical, held by a horizontal force P N, as shown. $\tan\alpha = \frac{8}{15}$.

Take $g = 9.8$ m s^{-2}.

Find the tension in the string and the value of P.

$\cos\alpha = 15/17$, $\sin\alpha = 8/17$

Vertical: $T\cos\alpha = mg = 58.8$, so $T = 58.8 \times 17/15$

Horizontal: $P = T\sin\alpha = 66.64 \times 8/17$

Answer: T = 66.64 N; P = 31.36 N

2 Expectation and variance of aX + b (AQA, OCR, MEI, Eduqas, CIE)

The random variable X has $E(X) = 8$ and $\text{Var}(X) = 2$.

Find (a) $E(4X - 1)$,

(b) $\text{Var}(4X - 1)$, (c) $E(X^2)$.

(a) $E(aX + b) = aE(X) + b = 4 \times 8 - 1 = 31$

(b) $\text{Var}(aX + b) = a^2\text{Var}(X) = 16 \times 2 = 32$

(c) $\text{Var}(X) = E(X^2) - [E(X)]^2$, so $E(X^2) = 2 + 64 = 66$

Answer: (a) 31 (b) 32 (c) 66

3 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $\frac{(2x^2)^2 \times 5x^4}{x^6}$.

$(2x^2)^2 = 4x^4$

Numerator $= 4 \times 5x^8 = 20x^8$

Divide: $\frac{20}{1} = 20$ and $x^8 \div x^6 = x^2$

Answer: $20x^2$

4 Inverse trigonometric functions (AQA, Edexcel, OCR, MEI, Eduqas)

Find the exact value of $\arccos(\cos\frac{2\pi}{3})$.

$\cos\frac{2\pi}{3} = -\frac{1}{2}$

$\arccos(-\frac{1}{2})$ must lie in $[0, \pi]$, so the value is $\frac{2\pi}{3}$

Answer: $\frac{2\pi}{3}$

5 Two particles over a pulley (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles of mass 3 kg and 12 kg hang on a light string over a smooth fixed pulley, as shown, and are released from rest.

Take $g = 9.8$ m s^{-2}.

Find the acceleration and the tension in the string.

Heavier: $12g - T = 12a$

Lighter: $T - 3g = 3a$

Add: $9g = 15a$, so $a = 5.88$

$T = 3(9.8 + 5.88)$

Answer: a = 5.88 m s^{-2}; T = 47.04 N

6 Perpendicular lines and shortest distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line l has equation $y = 2x + 5$. The point P is $(3, -4)$.
(a) Find the line through P perpendicular to l, as $ax + by + c = 0$ with integer a, b, c.
(b) Find the foot F and the exact distance PF.

(a) Gradient of l = 2, so the perpendicular has gradient $-\frac{1}{2}$

$y + 4 = -\frac{1}{2}(x - 3)$, giving $x + 2y + 5 = 0$

(b) Solve $y = 2x + 5$ with $x + 2y + 5 = 0$: F = $(-3, -1)$

$PF^2 = 6^2 + (-3)^2 = 45$

Answer: (a) $x + 2y + 5 = 0$ (b) F$(-3, -1)$, PF $= 3\sqrt{5}$

7 Arc length and sector area (radians) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has area 11.25 cm^2 and angle 0.9 radians.
Find (a) the radius, (b) the perimeter of the sector.

$\frac{1}{2}r^2\theta = 11.25$, so $r^2 = \frac{2 \times 11.25}{0.9} = 25$, $r = 5$

Perimeter $= 2r + r\theta = 10 + 4.5 = 14.5$ cm

Answer: (a) 5 cm (b) 14.5 cm

8 Finding mean and standard deviation from two probabilities (All boards)

The random variable X is normally distributed.
$P(X < 30) = 0.1587$ and $P(X > 60) = 0.0228$.
Find the mean and standard deviation (3 s.f.).

$z_1 = -0.9998$ and $z_2 = 1.999$

$30 = \mu + (-0.9998)\sigma$ and $60 = \mu + 1.999\sigma$

Subtract: $30 = 2.999\ \sigma$, so $\sigma = 10.0$

$\mu = 30 - (-0.9998)(10.00) = 40.0$

Answer: $\mu = 40.0$, $\sigma = 10.0$

9 Coefficient in a product with a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^2 in the expansion of
$(1 + x)(2 - 2x)^6$.

$(2 - 2x)^6$ has x^1 coefficient -384 and x^2 coefficient 960

x^2 term of the product $= 1 \times 960 + 1 \times (-384)$

$= 576$

Answer: 576

10 Compound angle formulae (exact values) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sin A = \frac{7}{25}$ and $\sin B = \frac{9}{41}$, where A and B are acute,
find the exact value of $\sin(A + B)$.

$\cos A = \frac{24}{25}$ and $\cos B = \frac{40}{41}$

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

$= \frac{7}{25} \times \frac{40}{41} + \frac{24}{25} \times \frac{9}{41} = \frac{496}{1025}$

Answer: $\frac{496}{1025}$