Day 237 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A company employs 100 managers, 260 technicians and 160 operators.
A stratified sample of size 26 is taken. Find the number selected from each group.

Total = 100 + 260 + 160 = 520

Sampling fraction = 26/520 = $\frac{1}{20}$

managers: 100 ÷ 20 = 5
technicians: 260 ÷ 20 = 13
operators: 160 ÷ 20 = 8
Answer: managers 5, technicians 13, operators 8

2 Exponential quadratics and ln equations (All boards)

Solve ln(5x + 5) = 5, giving your answer to 3 s.f.

$5x + 5 = e^5$

$x = \frac{e^5 - 5}{5} = 28.68$

Answer: x = 28.7

3 Will it move? Limiting friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of mass 20 kg rests on a rough horizontal floor, with coefficient of friction 0.6. A horizontal force of 94 N is applied to the box.
Take g = 9.8 m s^{-2}.
Determine whether the box moves. Find the friction force, or the acceleration if it moves.

R = 20 × 9.8 = 196, maximum friction μR = 0.6 × 196 = 117.6
94 < 117.6 so the box stays at rest
Friction balances the applied force: F = 94
Answer: Does not move; friction = 94 N

4 R cos(θ − α) form: express and maximum (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express cos θ + 3 sin θ in the form R cos(θ − α), where R > 0 and 0 < α < 90°. Give α to 1 d.p.

R cos α = 1, R sin α = 3

$R = \sqrt{1^2 + 3^2} = \sqrt{10}$

tan α = $\frac{3}{1}$, so α = 71.6°

Answer: $\sqrt{10}\cos(\theta - 71.6°)$

5 Smooth inclined plane (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle slides from rest down a line of greatest slope of a smooth plane inclined at α, where sin α = $\frac{3}{5}$ to the horizontal.
Take g = 9.8 m s^{-2}.
Find its acceleration, and the distance it travels in 2 s.

Along the plane: mg sin α = ma, so a = 9.8 × 0.6 = 5.88

$s = \frac{1}{2}at^2 = \frac{1}{2} \times 5.88 \times 2^2$

Answer: 5.88 m s^{-2}; 11.76 m

6 Quadratic equations in sin x, cos x, tan x (All boards)

Solve $\tan^2 x + \tan x - 2 = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p. where necessary.

(tan x − 1)(tan x + 2) = 0, so tan x = 1 or tan x = −2
tan x = 1: x = 45.0°, 225.0°
tan x = −2: x = 116.6°, 296.6°
Answer: x = 45.0°, 116.6°, 225.0°, 296.6°

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7 Coefficient in a product with a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^2 in the expansion of
$(1 + 4x)(2 + x)^7$.

$(2 + x)^7$ has x^1 coefficient 448 and x^2 coefficient 672
x^2 term of the product = 1 × 672 + 4 × 448
= 2464
Answer: 2464

8 Normal approximation to the binomial (AQA, OCR, CIE)

X ~ B(60, 0.5). Use a normal approximation to find P(X ≥ 25), to 3 significant figures.

Y ~ N(np, np(1 − p)) = N(30.00, 15.00), so σ = $\sqrt{15.00}$ = 3.873
Use a continuity correction.
P(X ≥ 25) ≈ P(Y > 24.5) = P(Z > −1.420) = 0.922
Answer: 0.922

9 Tangent lines of a given gradient (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line 4x − 3y = k is a tangent to the circle
$(x + 1)^2 + (y + 2)^2 = 25$.
Find the two possible values of k.

The radius to the point of contact is perpendicular to the line, so it is parallel to (4, −3).
A unit step in this direction is (4, −3)/5, so the points of contact are
(−1, −2) ± 5 × (4, −3)/5 = (−1, −2) ± (4, −3)
T = (3, −5) or (−5, 1), and k = 4x − 3y gives:
k = 27 or k = −23
Answer: k = 27 or k = −23

10 Equation of a circle from given information (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has centre (3, 1) and passes through the point (−5, 3).
Find an equation of the circle.

$r^2 = (−5 − 3)^2 + (3 − 1)^2$
= 64 + 4 = 68
Answer: $(x − 3)^2 + (y − 1)^2 = 68$