Day 236 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Simplifying surds (AQA, Edexcel, OCR, MEI, Eduqas)

Simplify $\frac{3}{\sqrt{3}} - \sqrt{12}$, giving your answer in the form $k\sqrt{3}$.

$\frac{3}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = 1\sqrt{3}$

$\sqrt{12} = 2\sqrt{3}$

$1\sqrt{3} - 2\sqrt{3} = -\sqrt{3}$

Answer: $-\sqrt{3}$

2 Projectile: speed and direction at time t (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is projected horizontally at 78.4 m s^{-1} from a point high above the ground.

Take g = 9.8 m s^{-2}.

Find the speed and the direction of motion 15 s later, to 1 d.p. below the horizontal.

Vertical speed = $9.8 \times 15 = 147$

Speed = $\sqrt{(78.4^2 + 147^2)} = 166.6$

$\tan \alpha = 147 \div 78.4$, $\alpha = 61.9°$

Answer: 166.6 m s^{-1} at 61.9° below horizontal

3 Solving index equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $25^{-2x-3} = 5^{-3x+1}$.

Write both sides as powers of 5: $5^{2(-2x-3)} = 5^{-3x+1}$

Equate the indices: $-4x - 6 = -3x + 1$

$-x = 7$

$x = -7$

Answer: $x = -7$

4 Regression line (AQA, Edexcel, OCR, MEI, Eduqas)

For 13 pairs of values, $\bar{x} = 8$, $\bar{y} = 25$, $S_{xx} = 20$ and $S_{xy} = -20$.

(a) Find the regression line of y on x in the form $y = a + bx$.

(b) Estimate y when x = 12.

(a) $b = \frac{S_{xy}}{S_{xx}} = \frac{-20}{20} = -1$

$a = \bar{y} - b\bar{x} = 25 - -1 \times 8 = 33$

$y = 33 - x$

(b) $y = 33 + (-1) \times 12 = 21$

Answer: $y = 33 - x$; $y = 21$

5 Perpendicular bisector (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is the point (0, −6) and B is the point (−6, 8).

Find the perpendicular bisector of AB as $ax + by + c = 0$, with integer a, b, c.

Midpoint of AB = (−3, 1)

Gradient of AB = $\frac{14}{-6} = -\frac{7}{3}$

Perpendicular gradient = $\frac{3}{7}$

$y - 1 = \frac{3}{7}(x + 3)$, which gives $3x - 7y + 16 = 0$

Answer: $3x - 7y + 16 = 0$

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6 Integrating trigonometric functions using identities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact value of

$\int_0^{\frac{\pi}{4}} (\cos^2 2x + 1 - \sin^2 2x)\, dx$.

Use $\cos^2 2x - \sin^2 2x = \cos 4x$

$\int (\cos^2 2x + 1 - \sin^2 2x)\, dx = x + \frac{\sin 4x}{4}$

$= [x + \frac{\sin 4x}{4}]_0^{\frac{\pi}{4}}$

$= \frac{\pi}{4}$

Answer: $\boldsymbol{\frac{\pi}{4}}$

7 Two independent binomials (All boards)

X ~ B(9, 0.5) and Y ~ B(10, 0.3) are independent.
Find P(X = 3 and Y ≤ 2), to 3 significant figures.

P(X = 3) = 0.1641
P(Y ≤ 2) = 0.3828
Independent, so multiply: 0.1641 × 0.3828 = 0.0628
Answer: 0.0628

8 Rod hinged at one end and held by a string (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 4 m and mass 18 kg is hinged at A and held horizontal by a light string attached at B.

The string makes an angle θ with the rod, where $\sin\theta = \frac{3}{5}$. Take g = 9.8 m s^{-2}. Find the tension in the string.

Moments about A: T sin θ × 4 = mg × 2

$T \times \frac{3}{5} \times 4 = 176.4 \times 2$

T = 352.8 ÷ 2.4
Answer: 147 N

9 Equations using $\sin^2 x + \cos^2 x = 1$ (All boards)

Solve $8 \sin x + 3 \cos^2 x - 7 = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p. where necessary.

Use $\cos^2 x = 1 - \sin^2 x$: $8 \sin x - 3 \sin^2 x - 4 = 0$, i.e. $3 \sin^2 x + 4 - 8 \sin x = 0$

$(3 \sin x - 2)(\sin x - 2) = 0$, so $\sin x = \frac{2}{3}$ or 2

$\sin x = \frac{2}{3}$: x = 41.8°, 138.2°

sin x = 2 has no solutions
Answer: x = 41.8°, 138.2°

10 Parametric differentiation: tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations

$x = 2t + 1$, $y = t^2 - 4$.

Find the gradient at $t = -2$ and the equation of the normal there.

$\frac{dx}{dt} = 2$, $\frac{dy}{dt} = 2t$

Gradient $= \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = -2$ at $t = -2$

Point: $(-3, 0)$; normal gradient $= \frac{1}{2}$

$y = \frac{1}{2}(x + 3)$, so $y = \frac{1}{2}x + \frac{3}{2}$

Answer: gradient -2; normal $y = \frac{1}{2}x + \frac{3}{2}$