Day 235 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 3D vectors: magnitude, distance and unit vectors (All boards)

Points A and B have position vectors $\begin{pmatrix}2\\-3\\2\end{pmatrix}$ and $\begin{pmatrix}5\\3\\4\end{pmatrix}$.

Find AB and the distance AB.

$\text{AB} = \text{OB} - \text{OA} = \begin{pmatrix}3\\6\\2\end{pmatrix}$

$|\text{AB}| = \sqrt{3^2 + 6^2 + 2^2} = \sqrt{49} = 7$

Answer: AB $= \begin{pmatrix}\mathbf{3}\\\mathbf{6}\\\mathbf{2}\end{pmatrix}$**, distance 7**

2 Constant acceleration (suvat) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car moves in a straight line with constant acceleration. Its initial speed is 2 m s^{-1}, its final speed is 18.5 m s^{-1} and it travels 112.75 m.

Find the acceleration.

$v^2 = u^2 + 2as$

$18.5^2 = 2^2 + 2 \times a \times 112.75$

$338.25 = 225.5\,a$

Answer: 1.5 m $\mathbf{s^{-2}}$

3 Stationary points of exponential and log curves (All boards)

Find the coordinates of the stationary point of the curve $y = xe^{2x}$,
and determine its nature.

$\frac{dy}{dx} = (2x + 1)e^{2x} = 0$

$x = -\frac{1}{2}$

$\frac{d^2y}{dx^2} = 4(x + 1)e^{2x}$

At $x = -\frac{1}{2}$: $\frac{d^2y}{dx^2} = \frac{2}{e} > 0$, so a minimum

Answer: $\mathbf{\left(-\frac{1}{2}, -\frac{1}{2e}\right)}$ **minimum**

4 Differentiating displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves on a straight line. Its displacement from O at time t seconds is $s = t^3 + 6t$ metres.

Find its velocity and acceleration when t = 3.

$v = ds/dt = 3t^2 + 6$

$a = dv/dt = 6t$

$t = 3$: $v = 33$, $a = 18$

Answer: v = 33 m $\mathbf{s^{-1}}$**, a = 18 m** $\mathbf{s^{-2}}$

5 Intersection of a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations

$2x + y = 5$

$x^2 + y^2 = 185$

$y = -2x + 5$; substitute: $x^2 + (-2x + 5)^2 = 185$

$5x^2 - 20x - 160 = 0$, so $x^2 - 4x - 32 = 0$

$(x + 4)(x - 8) = 0$, so $x = -4$ or $x = 8$

$y = 5 - 2x$ gives $y = 13$ and $y = -11$

Answer: x = $\mathbf{-4}$**, y = 13 and x = 8, y =** $\mathbf{-11}$

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6 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas)

For small θ (in radians), use the small angle approximations

to find the value of $\lim_{\theta \to 0} \frac{\tan 4\theta \sin \theta}{1 - \cos 3\theta}$.

$\tan 4\theta \sin \theta \approx 4\theta \times \theta = 4\theta^2$

$1 - \cos 3\theta \approx \frac{9\theta^2}{2}$

Ratio = $\frac{4\theta^2}{\frac{9\theta^2}{2}} = \frac{8}{9}$

Answer: $\mathbf{\frac{8}{9}}$

7 Geometric series and logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has first term 4 and common ratio 3.

Find the smallest value of n for which the sum of the first n terms exceeds 20000.

$S_n = \frac{4(3^n - 1)}{2} > 20000$

$3^n > 10001$

$n > \frac{\lg 10001}{\lg 3} = 8.384$

So the smallest integer n is 9

Answer: n = 9

8 Poisson: changing the interval (CIE)

Meteors are seen at random at an average rate of 2.5 per hour.

Find the probability that in 120 minutes there are exactly 1 (3 s.f.).

Rate in 120 minutes: $\lambda = 2.5 \times \frac{120}{60} = 5$

X ~ Po(5)

$P(X = 1) = e^{-5} \times \frac{5^1}{1!} = 0.0337$

Answer: 0.0337

9 Algebraic division (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the quotient and the remainder when

$3x^3 + 3x^2 - 2x + 4$ is divided by (x + 1).

$3x^2 \times (x + 1) = 3x^3 + 3x^2$, leaving $-2x + 4$

$-2 \times (x + 1) = -2x - 2$, leaving 6

Quotient $3x^2 - 2$, remainder 6

Answer: quotient $3x^2 - 2$, remainder 6

10 Binomial two-tailed test (All boards)

The probability that a student passes a driving test first time is thought to be 0.5. In a sample of 40 students, 15 pass first time.

Test, at the 5% significance level, whether the probability is different from 0.5.

H_0: p = 0.5, H_1: p ≠ 0.5. Under H_0, X ~ B(40, 0.5)

$P(X \leq 15) = 0.0769$

Two-tailed: compare with 0.025. 0.0769 > 0.025, so do not reject H_0

There is insufficient evidence that the probability is different from 0.5

Answer: do not reject H_0: insufficient evidence that the probability is different from 0.5