Day 234 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 3D vectors: magnitude, distance and unit vectors (All boards)

The vector $\mathbf{a} = \begin{pmatrix} 2 \\ 6 \\ 9 \end{pmatrix}$.

Find $|\mathbf{a}|$ and a unit vector in the direction of $\mathbf{a}$.

$|\mathbf{a}| = \sqrt{2^2 + 6^2 + 9^2} = \sqrt{121} = 11$

Unit vector $= \frac{1}{11}\mathbf{a}$

Answer: $|\mathbf{a}| = 11$, unit vector $\frac{1}{11}\begin{pmatrix} 2 \\ 6 \\ 9 \end{pmatrix}$

2 Describing transformations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe the single transformation that maps the graph of $y = f(x)$ onto the graph of:

(a) $y = f(\frac{x}{4})$

(b) $y = 2f(x)$

(c) $y = f(-x)$

(a) $y = f(\frac{x}{4})$ is a stretch, scale factor 4, parallel to the x-axis.

(b) $y = 2f(x)$ is a stretch, scale factor 2, parallel to the y-axis.

(c) $y = f(-x)$ is a reflection in the y-axis.

Answer: (a) stretch, scale factor 4, parallel to the x-axis (b) stretch, scale factor 2, parallel to the y-axis (c) reflection in the y

3 Conditional probability from a count (All boards)

80 customers: 24 buy coffee, 13 buy tea, 49 do neither.

(a) How many do both?

(b) Given a person who buys tea, find P(this person buys coffee).

(a) At least one: $80 - 49 = 31$. Both $= 24 + 13 - 31 = 6$

(b) Of the 13 who buy tea, 6 also do the other: $P = \frac{6}{13} = \frac{6}{13}$

Answer: (a) 6 (b) $\frac{6}{13}$

4 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $x^2 + 2xy + 2x + 3y^2 + 4y = 3$.

Find $\frac{dy}{dx}$ in terms of x and y, and the gradient at $(-3, 0)$.

Differentiate with respect to x: $2x\frac{dy}{dx} + 2x + 6y\frac{dy}{dx} + 2y + 4\frac{dy}{dx} + 2 = 0$

$\frac{dy}{dx} = \frac{-x - y - 1}{x + 3y + 2}$

At $(-3, 0)$: gradient $= -2$

Answer: $\frac{dy}{dx} = \frac{-x - y - 1}{x + 3y + 2}$; gradient -2

5 Beam about to tilt (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform plank AB of length 6 m and mass 30 kg rests horizontally on supports at C and D, where AC = 0.5 m and DB = 2 m.

A girl of mass 15 kg walks from D towards B. Find how far she is from D when the plank is about to tilt.

About to tilt about D: reaction at C is zero

Moments about D: $30\text{ g} \times (4 - 3) = 15\text{ g} \times x$

$30 \times 1 = 15x$

Answer: 2 m

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6 Grouped data: mean and median (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table summarises 36 measurements of m.
(a) Estimate the mean.
(b) Estimate the median using linear interpolation.

(a) Using mid-points: $\Sigma fx = 2340$, so mean = $2340 \div 36 = 65.0$

(b) $\frac{n}{2} = 18$. Cumulative frequency before the median class = 11

Median class 60 to 80. Median = $60 + \frac{7}{11} \times 20 = 72.7$

Answer: mean ≈ 65.0, median ≈ 72.7

7 Vertical projection upwards (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is projected vertically upwards with speed 21 m s^{-1} from ground level.
Take g = 9.8 m s^{-2}. Find, to 3 significant figures,
(a) the greatest height reached, (b) the time of flight.

(a) $0 = 21^2 - 2 \times 9.8 \times H$, so $H = 441 \div 19.6$
(b) s = 0: $t = 2 \times 21 \div 9.8$
Answer: (a) 22.5 m (b) 4.29 s

8 Product rule (All boards)

Given that $y = x^2e^{-x}$,
find $\frac{dy}{dx}$, and its value when x = 1.

Product rule with $u = x^2$, $v = e^{-x}$
$u' = 2x$, $v' = -e^{-x}$
$\frac{dy}{dx} = u'v + uv' = -x(x - 2)e^{-x}$
When x = 1: $\frac{dy}{dx} = \frac{1}{e}$
Answer: $\frac{dy}{dx} = -x(x - 2)e^{-x}$; value $\frac{1}{e}$

9 Quadratics in a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the equation $5^{2x} - 126 \times 5^x + 125 = 0$.

Let $y = 5^x$. Then $5^{2x} = y^2$, so $y^2 - 126y + 125 = 0$
$(y - 125)(y - 1) = 0$
y = 125 or y = 1
$5^x = 125$ gives x = 3; $5^x = 1$ gives x = 0.
Answer: x = 0 or x = 3

10 Intersection of a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = x^2 - 4x + 3$ and the line $y = -4x + 12$ meet at the points A and B.
Find the coordinates of A and B, and the exact length AB.

$x^2 - 4x + 3 = -4x + 12$
$x^2 - 9 = 0$, so $(x + 3)(x - 3) = 0$
x = −3 or x = 3
$y = -4x + 12$: y = 24 and y = 0
$AB^2 = 6^2 + -24^2 = 612$
Answer: A(−3, 24), B(3, 0), AB = $6\sqrt{17}$