**Day 233** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 1 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $2x^2 - 20x + 58$ in the form $a(x + p)^2 + q$.
(b) Write down the coordinates of the minimum point of $y = 2x^2 - 20x + 58$.

Take out the factor 2: $2x^2 - 20x + 58 = 2(x^2 - 10x) + 58$
$= 2[(x - 5)^2 - 25] + 58$
$= 2(x - 5)^2 + 8$
$(x - 5)^2 \geq 0$ with equality at $x = 5$, so the minimum is 8 at $x = 5$.
**Answer: (a) $2(x - 5)^2 + 8$ (b) (5, 8)**

## 2 Locating roots: change of sign (All boards)

The function $f(x) = \ln x + x - 3$.
Show that $f(x) = 0$ has a root between $x = 2$ and $x = 2.5$.

$f(2) = -0.307$
$f(2.5) = 0.416$
f is continuous and changes sign, so there is a root in the interval
**Answer: f(2) < 0 and f(2.5) > 0: sign change**

## 3 Independent and mutually exclusive events (All boards)

A and B are mutually exclusive events with $P(A) = 0.2$ and $P(A \cup B) = 0.45$.
(a) Find P(B).
(b) Find $P(A' \cap B')$.

(a) Mutually exclusive: $P(A \cup B) = P(A) + P(B)$
$P(B) = 0.45 - 0.2 = 0.25$
(b) $P(A' \cap B') = 1 - P(A \cup B) = 0.55$
**Answer: (a) 0.25 (b) 0.55**

## 4 Proof using parity (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $n^3 + 5n + 2$ is even for all integers n.

$n^3 + 5n = n(n^2 + 5)$
If n is even, $n(n^2 + 5)$ is even.
If n is odd, $n^2$ is odd and $n^2 + 5$ is even, so $n(n^2 + 5)$ is even.
So $n^3 + 5n$ is always even, and adding 2 gives an even number.
**Answer: expression is even**

## 5 Displacement from a v-t formula; average velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves along a straight line so that its displacement from O after t s is $s = 2t^3 + 2t^2$ metres.
Find (a) the average velocity over the first 3 seconds, (b) the velocity at $t = 3$.

(a) $s(3) = 72$, so average velocity $= 72 \div 3$
(b) $v = ds/dt = 6t^2 + 4t$, $v(3) = 66$
**Answer: (a) 24 m s$^{-1}$ (b) 66 m s$^{-1}$**

## 6 Perpendicular lines and shortest distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line l has equation $y = -\frac{3}{2}x - 6$. The point P is $(1, -1)$.

(a) Find the line through P perpendicular to l, as $ax + by + c = 0$ with integer a, b, c.
(b) Find the foot F and the exact distance PF.

(a) Gradient of l $= -\frac{3}{2}$, so the perpendicular has gradient $\frac{2}{3}$

$y + 1 = \frac{2}{3}(x - 1)$, giving $2x - 3y - 5 = 0$

(b) Solve $y = -\frac{3}{2}x - 6$ with $2x - 3y - 5 = 0$: $F = (-2, -3)$

$PF^2 = 3^2 + 2^2 = 13$
**Answer: (a) $2x - 3y - 5 = 0$ (b) F(−2, −3), PF = $\sqrt{13}$**

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## 7 Derivatives of tan, sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Differentiate y = sec 3x with respect to x,

and find the exact gradient when $x = \frac{\pi}{9}$.

Use the standard results for tan, sec, cosec and cot with the chain rule

$\frac{dy}{dx} = 3 \tan 3x \sec 3x$

At $x = \frac{\pi}{9}$: $\frac{dy}{dx} = 6\sqrt{3}$

**Answer: $\frac{dy}{dx} = 3 \tan 3x \sec 3x$; gradient $6\sqrt{3}$**

## 8 Vertical projection upwards (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A tennis ball is projected vertically upwards with speed 18 m $s^{-1}$ from ground level.
Take g = 9.8 m $s^{-2}$. Find, to 3 significant figures,
(a) the greatest height reached, (b) the time of flight.

(a) $0 = 18^2 - 2 \times 9.8 \times H$, so $H = 324 \div 19.6$
(b) s = 0: $t = 2 \times 18 \div 9.8$
**Answer: (a) 16.5 m (b) 3.67 s**

## 9 Binomial: smallest n (All boards)

Given that each component fails independently with probability 0.05, find the least number of components needed for the probability of at least one success to exceed 0.99.

P(at least one) = $1 - 0.95^n > 0.99$
$0.95^n < 0.01$
$n > \frac{\log 0.01}{\log 0.95} = 89.78$
**Answer: n = 90**

## 10 Equations with sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\cot^2 x + 5 \operatorname{cosec} x + 7 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p.

Use $\cot^2 x = \operatorname{cosec}^2 x - 1$: $\operatorname{cosec}^2 x + 5 \operatorname{cosec} x + 6 = 0$
(cosec x + 2)(cosec x + 3) = 0, so cosec x = −2 or −3
cosec x = −2, so $\sin x = -\frac{1}{2}$: x = 210.0°, 330.0°
cosec x = −3, so $\sin x = -\frac{1}{3}$: x = 199.5°, 340.5°
**Answer: x = 199.5°, 210.0°, 330.0°, 340.5°**