Day 232 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Interpreting regression (AQA, Edexcel, OCR, MEI, Eduqas)

Regression line (x from 2 to 10): $y = 3.2 + 0.8x$.
x is the advertising spend, y the weekly sales.
(a) Interpret the gradient.
(b) Estimate y when x = 13 and comment.

(a) The gradient: for each extra £100 of advertising, y is predicted to increase by 0.8
(b) $y = 3.2 + (0.8) \times 13 = 13.6$
x = 13 lies outside the range of the data (extrapolation), so the estimate may be unreliable
Answer: (a) y increases by 0.8 per extra £100 of advertising (b) 13.6, unreliable (extrapolation)

2 Standard integrals (definite) (All boards)

Find the exact value of

$\int_0^{\frac{\pi}{2}} (6 \sin x + \cos x)\, dx$.

$\int (6 \sin x + \cos x)\, dx = \sin x - 6 \cos x$

$= [\sin x - 6 \cos x]_0^{\frac{\pi}{2}}$
$= 7$
Answer: 7

3 Will it move? Limiting friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of mass 8 kg rests on a rough horizontal floor, with coefficient of friction 0.25. A horizontal force of 24 N is applied to the box.
Take $g = 9.8$ m s^{-2}.
Determine whether the box moves. Find the friction force, or the acceleration if it moves.

$R = 8 \times 9.8 = 78.4$, maximum friction $\mu R = 0.25 \times 78.4 = 19.6$
$24 > 19.6$ so the box moves, $F = 19.6$
$24 - 19.6 = 8a$
Answer: It moves; $a = 0.55$ m s^{-2}

4 Chain rule (All boards)

Given that $y = \frac{1}{(5x + 4)^2}$,

find $\frac{dy}{dx}$, and the gradient of the curve when x = 0.

Chain rule: differentiate the outer function, then multiply by the derivative of the inner function
$\frac{dy}{dx} = -\frac{10}{(5x + 4)^3}$

When x = 0: gradient $= -\frac{5}{32}$

Answer: $\frac{dy}{dx} = -\frac{10}{(5x + 4)^3}$; gradient $-\frac{5}{32}$

5 Regression line (AQA, Edexcel, OCR, MEI, Eduqas)

For 10 pairs of values, $\bar{x} = 5$, $\bar{y} = 25$, $S_{xx} = 40$ and $S_{xy} = -80$.
(a) Find the regression line of y on x in the form $y = a + bx$.
(b) Estimate y when x = 2.

(a) $b = \frac{S_{xy}}{S_{xx}} = \frac{-80}{40} = -2$

$a = \bar{y} - b\bar{x} = 25 - -2 \times 5 = 35$
$y = 35 - 2x$
(b) $y = 35 + (-2) \times 2 = 31$
Answer: $y = 35 - 2x$; $y = 31$

6 Parametric differentiation: tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 2\sin t$, $y = 5\cos 2t$.

Find $\frac{dy}{dx}$ in terms of t, and its exact value when $t = \frac{\pi}{4}$.

$\frac{dx}{dt} = 2\cos t$, $\frac{dy}{dt} = -10\sin 2t$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = -\frac{5\sin 2t}{\cos t}$

At $t = \frac{\pi}{4}$: $\frac{dy}{dx} = -5\sqrt{2}$

Answer: $\frac{dy}{dx} = -\frac{5\sin 2t}{\cos t}$; value $-5\sqrt{2}$

7 Reciprocal graph: asymptotes and intercepts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $y = \frac{-1}{x-4} + 3$.

(a) Write down the asymptotes.

(b) Find where the curve crosses the axes.

(a) The denominator is zero when $x = 4$, and $y \to 3$ as $x \to \pm\infty$

Asymptotes: $x = 4$ and $y = 3$

(b) $x = 0$: $y = \frac{1}{4} + 3 = \frac{13}{4}$

$y = 0$: $\frac{-1}{x-4} = -3$, so $x - 4 = \frac{1}{3}$, $x = \frac{13}{3}$

Answer: (a) $x = 4$, $y = 3$ (b) $(0, \frac{13}{4})$ and $(\frac{13}{3}, 0)$

8 Rough inclined plane: sliding down (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 4 kg is released from rest on a rough plane inclined at angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$.

The coefficient of friction is 0.125. Take $g = 9.8$ m s^{-2}. Find the acceleration down the plane, and the distance moved in 2 s.

$\cos\alpha = 4/5$, $\sin\alpha = 3/5$

$R = mg\cos\alpha$, $F = \mu R$

$ma = mg\sin\alpha - \mu mg\cos\alpha$, so $a = 9.8(3/5 - 0.125 \times 4/5)$

$s = \frac{1}{2}at^2 = \frac{1}{2} \times 4.9 \times 2^2$

Answer: 4.9 m s^{-2}; 9.8 m

9 Arithmetic sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of all the multiples of 11 that lie between 98 and 383.

First multiple = $11 \times 9 = 99$, last multiple = $11 \times 34 = 374$

This is an arithmetic series with $d = 11$ and 26 terms

$S = \frac{26}{2}(99 + 374) = 6149$

Answer: 6149

10 Proof by exhaustion (AQA, Edexcel, OCR, MEI, Eduqas)

Prove, by exhaustion, that a cube number cannot be written in the form $9k + 7$, where k is an integer.

Write the integer as $n = 9q + j$, where $j = 0, 1, ..., 8$.

n^3 leaves the same remainder as j^3 when divided by 9.

$j = 0, 1, 2, 3, 4, 5, 6, 7, 8$ give remainders 0, 1, 8, 0, 1, 8, 0, 1, 8.

The only possible remainders are 0, 1, 8, never 7.

Answer: possible remainders 0, 1, 8, never 7