Day 231 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $3x - 4 \leq -3x - 9$.

$6x \leq -5$

Divide by 6: $x \leq -\frac{5}{6}$

Answer: $x \leq -\frac{5}{6}$

2 Free fall from rest (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A coin is dropped from rest from the top of a cliff, 78.4 m above the ground.
Take $g = 9.8$ m s^{-2}.
Find (a) the time taken to reach the ground, (b) its speed on landing.

(a) $s = ut + \frac{1}{2}at^2$: $78.4 = \frac{1}{2} \times 9.8 \times t^2$
$t^2 = 78.4 \div 4.9 = 16$, so $t = 4$
(b) $v = u + at = 9.8 \times 4$
Answer: (a) 4 s (b) 39.2 m s^{-1}

3 Proof by deduction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $(n + 3)^2 - (n - 3)^2$ is a multiple of 12 for all integer values of n.

$(n + 3)^2 = n^2 + 6n + 9$
$(n - 3)^2 = n^2 - 6n + 9$
Difference $= 12n = 12 \times n$
n is an integer, so this is a multiple of 12 for every integer n.
Answer: difference = 12n, a multiple of 12

4 Normal curve and shaded area (All boards)

$X \sim N(60, 8^2)$. The shaded area on the diagram represents the probability that X lies between two values.
Find this probability to 3 significant figures.

Shaded area = $P(52.00 < X < 68.00)$
$= P(X < 68.00) - P(X < 52.00)$
$= 0.8413 - 0.1587 = 0.683$
Answer: 0.683

5 Exponential growth and decay models (All boards)

The mass of a radioactive sample, M grams, after t days is $M = 60\,e^{-0.2t}$.
Find (a) the half-life, (b) the rate of decrease in mass when $t = 5$, each to 3 s.f.

(a) $60\,e^{-0.2t} = 30$: $e^{-0.2t} = \frac{1}{2}$, so $t = \frac{\ln 2}{0.2} = 3.466$ days

(b) $\frac{dM}{dt} = -0.2 \times 60\,e^{-0.2t} = -12\,e^{-0.2t}$

At $t = 5$: $\frac{dM}{dt} = -12\,e^{-1} = -4.415$

so mass is decreasing at 4.41 g per day
Answer: (a) 3.47 days (b) 4.41 g/day

6 Two independent binomials (All boards)

$X \sim B(7, 0.5)$ and $Y \sim B(9, 0.1)$ are independent.
Find $P(X = 3 \text{ and } Y \leq 1)$, to 3 significant figures.

$P(X = 3) = 0.2734$
$P(Y \leq 1) = 0.7748$
Independent, so multiply: $0.2734 \times 0.7748 = 0.212$
Answer: 0.212

7 Arc length and sector area (radians) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sector OAB of a circle with centre O, radius 13 cm and angle 1.1 radians.
Find (a) the arc length AB, (b) the area of the sector.

Arc length = $r\theta = 13 \times 1.1 = 14.3$ cm

Area = $\frac{1}{2}r^2\theta = \frac{1}{2} \times 13^2 \times 1.1 = 92.95$ cm^2

Answer: (a) 14.3 cm (b) 92.95 cm^2

8 Partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{-2x+2}{(x+1)(x-3)}$ in partial fractions.

$\frac{-2x+2}{(x+1)(x-3)} \equiv \frac{A}{(x+1)} + \frac{B}{(x-3)}$, so $-2x + 2 \equiv A(x-3) + B(x+1)$

$x = -1$: $4 = A \times (-4)$, so $A = -1$

$x = 3$: $-4 = B \times 4$, so $B = -1$

Answer: $-\frac{1}{(x+1)} - \frac{1}{(x-3)}$

9 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix}8\\4\\2\end{pmatrix} + \lambda\begin{pmatrix}-2\\-2\\0\end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix}2\\-2\\2\end{pmatrix} + \mu\begin{pmatrix}1\\-2\\0\end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $8 + (-2)\lambda = 2 + 1\mu$

j: $4 + (-2)\lambda = -2 + (-2)\mu$

Solving i and j: $\lambda = 3$, $\mu = 0$

k check: $2 + 0(3) = 2$ and $2 + 0(0) = 2$, equal

Point: $\begin{pmatrix}2\\-2\\2\end{pmatrix}$

Answer: $\begin{pmatrix}2\\-2\\2\end{pmatrix}$

10 Rod hinged at one end and held by a string (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 3 m and mass 3 kg is hinged at A and held horizontal by a light string attached at B.
The string makes an angle θ with the rod, where $\sin\theta = \frac{3}{5}$. Take $g = 9.8$ m s^{-2}. Find the tension in the string.

Moments about A: $T \sin\theta \times 3 = mg \times 1.5$

$T \times \frac{3}{5} \times 3 = 29.4 \times 1.5$

$T = 44.1 \div 1.8$

Answer: 24.5 N