Day 230 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Binomial probabilities (All boards)

X ~ B(10, 0.5). Find, to 3 significant figures,
(a) P(X ≤ 5)
(b) P(X = 5)

(a) P(X ≤ 5) from the cumulative binomial function
(a) = 0.623
(b) P(X = 5) = $^{10}C_5 \times 0.5^5 \times 0.5^5 = 0.246$

Answer: (a) 0.623 (b) 0.246

2 Solving exponential equations using logarithms (All boards)

Solve $3^x = 23$, giving your answer to 3 s.f.

Take logs: x ln 3 = ln 23
$x = \frac{\ln 23}{\ln 3} = 2.854$

Answer: x = 2.85

3 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-2x^2 - 6x < 4$.

Rearrange: $-2x^2 - 6x - 4 < 0$
Factorise: $-2(x + 1)(x + 2) < 0$
Critical values: $x = -2$ and $x = -1$
Sketch the ∩-shaped parabola: $x < -2$ or $x > -1$
Answer: $x < -2$ or $x > -1$

4 Derivatives of tan, sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Differentiate y = 5 cosec x with respect to x,
and find the exact gradient when $x = \frac{\pi}{6}$.

Use the standard results for tan, sec, cosec and cot with the chain rule
$\frac{dy}{dx} = -5 \cot x \operatorname{cosec} x$
At $x = \frac{\pi}{6}$: $\frac{dy}{dx} = -10\sqrt{3}$

Answer: $\frac{dy}{dx} = -5 \cot x \operatorname{cosec} x$; gradient $-10\sqrt{3}$

5 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{8}{1 - \sqrt{5}}$ in the form $p + q\sqrt{5}$, where p and q are integers.

Multiply top and bottom by the conjugate $(1 + \sqrt{5})$:
Denominator: $(1 - \sqrt{5})(1 + \sqrt{5}) = 1^2 - 1^2 \times 5 = -4$
Numerator: $(8)(1 + \sqrt{5}) = 8 + 8\sqrt{5}$
So the fraction = $\frac{8 + 8\sqrt{5}}{-4} = -2 - 2\sqrt{5}$

Answer: $-2 - 2\sqrt{5}$

6 Exponential quadratics and ln equations (All boards)

Solve $e^{2x} - 16\, e^x + 60 = 0$,
giving your answers as exact values.

Let $y = e^x$: $y^2 - 16\, y + 60 = 0$
$(y - 6)(y - 10) = 0$, so $e^x = 6$ or $e^x = 10$
$x = \ln 6$ or $x = \ln 10$
Answer: x = ln 6, x = ln 10

Day 230 **A Level Maths: B Tier 10-a-day: Model Solutions**

7 Tangent to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle C has equation $(x - 3)^2 + (y + 2)^2 = 65$.
The point P(−5, −3) lies on C.
Find an equation of the tangent to C at P, in the form ax + by + c = 0 with integer a, b, c.

Centre of C = (3, −2). Gradient of radius CP = $\frac{-1}{-8} = \frac{1}{8}$

The tangent is perpendicular to the radius: gradient = −8
y + 3 = −8(x + 5)
Multiply out and collect terms: 8x + y + 43 = 0
Answer: 8x + y + 43 = 0

8 Reading a histogram (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The histogram shows the times taken by a group of people to complete a task.
Estimate the number of people who took between 50 and 83 minutes.

Frequency = frequency density × class width.
50 to 70: 5 × 20 = 100
70 to 80: 3 × 10 = 30
80 to 83: 6 × 3 = 18
Total = 148
Answer: 148

9 Velocity parallel to a direction (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves so that at time t s its velocity is $\mathbf{v} = (t^2 + 4)\mathbf{i} + (6t - 24)\mathbf{j}$ m s^{-1}.
Find the time when the particle is moving in the direction of **i**, and its speed then.

Parallel to **i**: **j** component = 0
6t − 24 = 0, so t = 4
$\mathbf{v} = (1 \times 4^2 + 4)\mathbf{i} = 20\mathbf{i}$
Answer: t = 4 s, speed 20 m s^{-1}

10 Rod suspended by two vertical strings (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 3 m and mass 6 kg hangs horizontally from two vertical strings attached 0.5 m from A and 0.5 m from B.
A particle of mass 3 kg is attached 1 m from A. Take g = 9.8 m s^{-2}. Find the tension in each string.

Moments about the string near A: $T_2(2) = 6\,g(1) + 3\,g(0.5)$
$T_2 = 36.75$
$T_1 + T_2 = 9\,g$ so $T_1 = 88.2 - 36.75$
Answer: Near A: 51.45 N; near B: 36.75 N