Day 23 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Chain rule (All boards)

Given that $y = (3x + 3)^6$,

find $\frac{dy}{dx}$, and the gradient of the curve when $x = 1$.

Chain rule: differentiate the outer function, then multiply by the derivative of the inner function

$\frac{dy}{dx} = 4374(x + 1)^5$

When $x = 1$: gradient = 139968

Answer: $\frac{dy}{dx} = 4374(x + 1)^5$; gradient 139968

2 Coded data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

10 values of x are coded using $y = \frac{(x - 50)}{5}$.

$\Sigma y = 5$ and $\Sigma y^2 = 131$.

Find the mean and standard deviation of x (3 s.f.).

Mean of $y = 5 \div 10 = 0.5000$

Mean of $x = 50 + 5 \times 0.5000 = 52.5$

Variance of $y = \frac{131}{10} - (0.5000)^2 = 12.85$

SD of y = 3.585, so SD of $x = 5 \times 3.585 = 17.9$

Answer: mean 52.5, standard deviation 17.9

3 Constant acceleration with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle has velocity $(5\mathbf{i} + 3\mathbf{j})$ m s^{-1} at time $t = 0$ and moves with constant acceleration $(3\mathbf{j})$ m s^{-2}.

Find its velocity and speed when $t = 3$.

$v = u + at = (5\mathbf{i} + 3\mathbf{j}) + 3(3\mathbf{j})$

$= 5\mathbf{i} + 12\mathbf{j}$

Speed $= \sqrt{(5^2 + 12^2)}$

Answer: $\mathbf{v} = (5\mathbf{i} + 12\mathbf{j})$ m s^{-1}; speed 13 m s^{-1}

4 Equation of a circle from given information (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line segment AB is a diameter of a circle, where A is $(-4, -3)$ and B is $(2, -9)$.

Find an equation of the circle in the form

$(x - a)^2 + (y - b)^2 = k$.

Centre = midpoint of AB = $(-1, -6)$

Radius$^2 = (-4 - (-1))^2 + (-3 - (-6))^2 = 18$

Answer: $(x + 1)^2 + (y + 6)^2 = 18$

5 Simplify using double angle formulae (All boards)

Simplify cosec 4x + cot 4x,

giving your answer as a single trigonometric term.

$\text{cosec } 4x + \cot 4x = \frac{1 + \cos 4x}{\sin 4x}$

$= \frac{2\cos^2 2x}{2 \sin 2x \cos 2x} = \cot 2x$

Answer: cot 2x

6 Distribution of the sample mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The mass, X g, of a loaf is normally distributed with mean 40 and s.d. 6.
A random sample of 4 is taken. Find the probability that the mean mass is more than 44.5 (3 s.f.).

$\overline{X}$ ~ N(40, $\frac{6^2}{4}$), standard deviation 3.000

$z = \frac{44.5 - 40}{3.000} = 1.500$

P = 0.0668

Answer: 0.0668

7 Exponential growth and decay models (All boards)

The mass of a radioactive sample, M grams, after t days is $M = 50\,e^{-0.05t}$.
Find (a) the half-life, (b) the rate of decrease in mass when t = 5, each to 3 s.f.

(a) $50\,e^{-0.05t} = 25$: $e^{-0.05t} = \frac{1}{2}$, so $t = \frac{\ln 2}{0.05} = 13.86$ days

(b) $\frac{dM}{dt} = -0.05 \times 50\,e^{-0.05t} = -2.5\,e^{-0.05t}$

At t = 5: $\frac{dM}{dt} = -2.5\,e^{-0.25} = -1.947$

so mass is decreasing at 1.95 g per day

Answer: (a) 13.9 days (b) 1.95 g/day

8 Rough table and a hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 2 kg lies on a rough horizontal table (coefficient of friction 0.3). It is joined by a light string over a smooth pulley to a particle of mass 5 kg hanging freely.
Take g = 9.8 m s^{-2}.
The system is released. Find the acceleration and the tension in the string.

Block: $T - 0.3 \times 2\,g = 2\,a$
Hanging: $5\,g - T = 5\,a$
Add: $5\,g - 0.6\,g = 7\,a$
a = 6.16
$T = 5(9.8 - 6.16)$

Answer: a = 6.16 m s^{-2}; T = 18.2 N

9 Binomial expansion (any power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(1 - x)^{-\frac{1}{2}}$ in ascending powers of x up to and including the term in x^3.
(b) State the range of values of x for which the expansion is valid.

$(1 + ax)^n = 1 + n(ax) + \frac{n(n-1)}{2!}(ax)^2 + \frac{n(n-1)(n-2)}{3!}(ax)^3 + \ldots$

Here $n = -\frac{1}{2}$ and a = −1:

coefficients: x: $\frac{1}{2}$, x^2: $\frac{3}{8}$, x^3: $\frac{5}{16}$

Valid when |x| < 1, that is |x| < 1

Answer: (a) $1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \ldots$ (b) $|x| < 1$

10 Cubic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $(x - 3)(x - 5)x \leq 0$.

Critical values: x = 0, x = 3, x = 5

The cubic has a positive x^3 term, so the graph goes from bottom left to top right and crosses the axis at each critical value.
Reading from the sketch: $x \leq 0$ or $3 \leq x \leq 5$

Answer: $x \leq 0$ or $3 \leq x \leq 5$